



MATHEMATICS KITS, PRACTICAL ACTIVITIES AND EXPERIMENTS USER GUIDE

**LOWER SECONDARY
(S1-S3)**

Kigali, 2022

© 2022 Rwanda Basic Education Board

All rights reserved.

This book is the property of the Government of Rwanda. Credit must be provided to REB when the content is quoted.

FOREWORD

Dear teacher,

Rwanda Basic Education Board (REB) is honoured to present the Mathematics practical activities and experiments user guide for Lower Secondary (S1-S3). This book will serve as a guide to competence-based teaching and learning to ensure consistency and coherence in the learning of Mathematics.

In this book, special attention was paid to practical activities/experiments that facilitate the learning process in which students can manipulate concrete materials, develop ideas and make new discoveries during activities carried out individually, in pairs/ small groups.

In competence-based curriculum, practical activities open students' mind and provide them with the opportunities to interact with the world, use available tools, collect data, and effectively model real life problems.

For efficient use of this booklet, your role as a teacher is to:

- Plan your lessons and prepare appropriate teaching materials.
- Organize groups for students considering the importance of social constructivism.
- Engage students through active learning methods.
- Provide supervised opportunities for students to develop different competences by giving tasks which enhance critical thinking, problem solving, research, creativity and innovation, communication, and cooperation.
- Support and facilitate the learning process by valuing students' contributions in the practical activities.
- Guide students towards the conclusion on the results of the experiments.
- Encourage individual, peer and group evaluation of the work done and use appropriate competence-based assessment approaches and methods.

To facilitate you in your teaching activities, the content of this guide is self-explanatory so that you can easily use it. It is divided in 3 parts:

The part 1 explains the structure of this guide and gives you the general introduction on the role of practical activities and experiments in the implementation of Competence Based Curriculum (CBC).

The part 2 gives, the list of Mathematics kit items.

The part 3 explains selected practical activities and how you can facilitate them in lessons.

Even though this guide contains practical activities/experiments, they are not enough; As expert and experienced teacher, you can guide students to carry out more practical activities using improvised teaching resources.

I wish to sincerely extend my appreciation to the people who contributed towards the development of this guide, particularly REB staff who organized the whole process from its inception. Special appreciation goes also to teachers and independent experts in education who supported the exercise throughout. Any comment or contribution would be welcome for the improvement of this book for next versions.

Dr. MBARUSHIMANA Nelson

Director General, REB

ACKNOWLEDGEMENT

I wish to express my appreciation to the people who played a major role in the development and the editing of the user guide for Mathematics practical activities and experiments for Lower Secondary (S1-S3). It would not have been successful without active participation of different education stakeholders.

I owe gratitude to Curriculum Officers, teachers and independent people whose efforts during the development of this teaching and learning resource were very much valuable.

Finally, my word of gratitude goes to RQBEHCD Project for its financial support towards improving the quality of education in Rwanda.

Joan MURUNGI

Head of CTLR Department

LIST OF ACRONYMS

CBC: Competence-based curriculum

ICT: Information Communication Technology

Lab: Laboratory

STEM: Science, Technology, Engineering and Mathematics

KBC: Knowledge Based Curriculum

SET: Science and Elementary Technology

SPIU: Single Project Implementation Unit.

RQBEHCD: Rwanda Quality Basic Education for Human Capital Development

TABLE OF CONTENTS

| | |
|--|------------|
| FOREWORD | iii |
| ACKNOWLEDGEMENT | v |
| LIST OF ACRONYMS | vi |
| TABLE OF CONTENTS | vii |
| PART 1: GENERAL INTRODUCTION | 1 |
| PART 2: LIST OF MATHEMATICS KIT ITEMS DISTRIBUTED IN SCHOOLS. | 12 |
| PART 3: PRACTICAL ACTIVITIES OR EXPERIMENTS PER GRADE | 18 |
| PRACTICAL ACTIVITIES FOR S1 | 19 |
| PRACTICAL ACTIVITY 1: Representation of a problem using a Venn diagram involving two sets | 19 |
| PRACTICAL ACTIVITY 2: Representing set operations in a Venn diagram involving 3 sets | 22 |
| PRACTICAL ACTIVITY 3: Exploring relations between sets, defining domain and range, and creating mappings | 25 |
| PRACTICAL ACTIVITY 4: Forming and measuring angles..... | 27 |
| PRACTICAL ACTIVITY 5: Verifying the relation among the angles formed by a transversal of parallel lines | 30 |
| PRACTICAL ACTIVITY/ EXPERIMENT 6: Exploring the angles of a triangle.. | 33 |
| PRACTICAL ACTIVITY/ EXPERIMENT 7: Forming a straight angle..... | 35 |
| PRACTICAL ACTIVITY/ EXPERIMENT 8: Exploring Angles at the point..... | 38 |
| PRACTICAL ACTIVITY/ EXPERIMENT 9: Forming different shapes on a geoboard and exploring their areas..... | 41 |
| PRACTICAL ACTIVITY 10: Verifying Euler’s rule by counting the numbers of faces and vertices for a solid | 45 |
| PRACTICAL ACTIVITY 11: Exploring the concept of surface area of a cuboid by using folded up nets..... | 47 |
| PRACTICAL ACTIVITY 12: Exploring the concept of surface area of a prism by using folded up nets..... | 50 |
| PRACTICAL ACTIVITY 13: Exploring the surface area of a pyramid by using folded up nets | 53 |
| PRACTICAL ACTIVITY 14: Exploring the concept of surface area of a cylinder by using folded up nets..... | 55 |

| | |
|--|-----------|
| PRACTICAL ACTIVITY/ EXPERIMENT 15: Formula of the volume of a cylinder | 58 |
| PRACTICAL ACTIVITY 16: Data organisation/ collection and frequency distribution table | 61 |
| PRACTICAL ACTIVITY 17: Constructing Bar chart of a discrete data using real material..... | 64 |
| PRACTICAL ACTIVITY 18: Constructing histogram and frequency polygon of a discrete data using real material | 67 |
| PRACTICAL ACTIVITY 19: Determining the probability of an event happening when tossing coins | 70 |
| PRACTICAL ACTIVITY 20: Determining the probability of an event happening when playing cards | 72 |
| PRACTICAL ACTIVITY 21: Determining the probability of events from throwing a die | 75 |
| PRACTICAL ACTIVITIES FOR S2..... | 79 |
| PRACTICAL ACTIVITY 1: Addition and subtraction of second order polynomials using algebraic tiles..... | 79 |
| PRACTICAL ACTIVITY/ EXPERIMENT 2: Quadratic identities | 85 |
| PRACTICAL ACTIVITY 3: Factorization of quadratic polynomial..... | 88 |
| PRACTICAL ACTIVITY/EXPERIMENT 4: Exploring the Midpoint theorem | 92 |
| PRACTICAL ACTIVITY/ EXPERIMENT 5: Explore Thales' theorem..... | 95 |
| PRACTICAL ACTIVITY/ EXPERIMENT 6: Exploring Pythagoras theorem..... | 97 |
| PRACTICAL ACTIVITY 7: Addition and subtraction of vectors | 100 |
| PRACTICAL ACTIVITY 8: Vertical projection | 104 |
| PRACTICAL ACTIVITY 9: Image of object under central symmetry and properties of central symmetry | 107 |
| PRACTICAL ACTIVITY 10: Image of an object under rotation | 110 |
| PRACTICAL ACTIVITY 11: Image of an object under translation and the related properties | 115 |
| PRACTICAL ACTIVITY 12: Image of a given object under a rotation after a translation | 119 |
| PRACTICAL ACTIVITY 13: Frequency distribution table and a related accurate pie chart for grouped data..... | 122 |
| PRACTICAL ACTIVITY 14: Exploring the likelihood of a situation to happen | 125 |

| | |
|---|------------|
| PRACTICAL ACTIVITY 15: Probability of the occurrence of a real-life situation using Venn diagrams..... | 128 |
| PRACTICAL ACTIVITY/ EXPERIMENT 16: Using a tree diagram to determine sample space, favourable outcomes, and probability that an event happens | 132 |
| PRACTICAL ACTIVITIES FOR S3..... | 137 |
| PRACTICAL ACTIVITY 1: Changing from base ten to base two. | 137 |
| PRACTICAL ACTIVITY 2: Factorization of quadratic trinomial | 140 |
| PRACTICAL ACTIVITY/ EXPERIMENT 3: Pythagoras theorem | 145 |
| PRACTICAL ACTIVITY/ EXPERIMENT 4: Median and the hypotenuse in a right triangle | 148 |
| PRACTICAL ACTIVITY/ EXPERIMENT 5: Median of a triangle and the surface area | 150 |
| PRACTICAL ACTIVITY 6: Meaning of different trigonometric ratios | 152 |
| PRACTICAL ACTIVITY 7: Trigonometric ratio of an angle and the variation of lengths of the sides of the triangle | 155 |
| PRACTICAL ACTIVITY 8: Standard trigonometric identity | 157 |
| PRACTICAL ACTIVITY/ EXPERIMENT 9: Area of a circle..... | 159 |
| PRACTICAL ACTIVITY/EXPERIMENT 10: Angle subtended at the centre of a circle by unequal chords..... | 161 |
| PRACTICAL CTIVITY/ EXPERIMENT 11: Angles subtended at the centre of a circle by equal chords..... | 163 |
| PRACTICAL ACTIVITY/ EXPERIMENT 12: Angle subtended at the centre and at any point on the circumference of circle..... | 165 |
| PRACTICAL ACTIVITY/ EXPERIMENT 13: Angles in the same segment of a circle | 167 |
| PRACTICAL ACTIVITY/ EXPERIMENT 14: The angle in a semi-circle | 169 |
| PRACTICAL ACTIVITY/ EXPERIMENT 15: Line through the centre of a circle bisecting a chord..... | 171 |
| PRACTICAL ACTIVITY 16: Centre of circular object..... | 173 |
| PRACTICAL ACTIVITY/ EXPERIMENT 17: Angles in a cyclic quadrilateral ... | 175 |
| PRACTICAL ACTIVITY/EXPERIMENT 18: Sum of a pair of opposite angles of a non-cyclic quadrilateral..... | 177 |
| PRACTICAL ACTIVITY 19: Similar polygons | 180 |

| | |
|---|------------|
| PRACTICAL ACTIVITY/ EXPERIMENT 20: Explore Thales' theorem..... | 182 |
| PRACTICAL ACTIVITY/ EXPERIMENT 21: Area of similar shapes | 184 |
| PRACTICAL ACTIVITY/ EXPERIMENT 22: Image of an object under an enlargement | 187 |
| PRACTICAL ACTIVITY/ EXPERIMENT 23: Scale factor for an enlargement. | 192 |
| PRACTICAL ACTIVITY 24: Explore the correlation between two variables.. | 195 |
| REFERENCE..... | 198 |
| Annex 1: Name of commonly hazard symbols useful in the laboratory..... | 199 |

PART 1: GENERAL INTRODUCTION

a. Structure of the user guide

The user guide for practical activities and experiments in S1-S3 Mathematics is divided in 3 parts:

The part 1 explains the structure of this book and gives you the general introduction on the role of practical activities and experiments in the implementation of CBC.

The part 2 gives the list of mathematics kit items.

The part 3 details the practical activities or laboratory experiments and how to facilitate them in lessons.

b. Practical activities in the Competence Based Curriculum

A competence-based curriculum (CBC) focuses on what learners can do and apply in different situations by developing skills, attitudes, and values in addition to knowledge and understanding. This learning process is learner-focused, where a learner is engaged in active and participatory learning activities, and learners finally build new knowledge from prior knowledge. Since 2015, the Rwanda Education system has changed from KBC to CBC for preparing students that meet the national and international job market requirements and job creation. Therefore, implementing the CBC education system necessitates qualitative laboratory practical works for Mathematics and Science as more highlighted aspects.

In addressing this necessity, practical activities/ experiments play a major role. A child is motivated to learn mathematics by getting involved in handling various concrete manipulatives in various activities. In addition to activities, games in Mathematics also help the child's involvement in learning by strategizing and reasoning.

For learning mathematical concepts through the above-mentioned approach, child-centred Mathematics kits have been availed for the students of lower Secondary schools. The kits include various kit items along with a manual for performing activities.

The kit broadly covers the activities in the areas of algebra, geometry, trigonometry, statistics and probability.

The kit has the following advantages:

- Availability of necessary and common materials in all schools,
- Multipurpose use of kit items,
- Economy of time in doing the activities,
- Portability from one place to another,

- Provision for teacher's innovation,
- Low-cost material and use of indigenous resources.

Apart from the kit, the user guide for Mathematics practical activities to be used by teachers was developed. This user guide is designed to help mathematics teachers to perform high-quality experiments for mathematics. This user guide structure induces learner's interest, achievement, and motivation through the qualitative mathematics experiments offered by their teachers and will finally lead to the targeted goals of the CBC, particularly in the field of Mathematics and Science.

In CBC, learners hand on the materials and reveal the theory behind the experiment done. Here, experiments are done inductively, where experiments serve as an insight towards revealing the theory. Thus, the experiment starts, and theory is produced from the results of the experiment.

Mathematics experiment is a procedure undertaken to demonstrate a known fact such as formula. We have in this book practical activities which are also experiments.

c. Type of practical works

Mathematics laboratory is a place where students can learn and explore mathematical concepts and verify mathematical facts and theorems through a variety of activities using different techniques.

The goal of experiment defines its type and how it is organized. In this version of Mathematics experiment user guide, we prefer to use practical work instead of laboratory experiment. Therefore, before doing practical work, it is important to have a clear idea of the objective.

The three types of practical works are:

1. **Equipment-based practical work:** the goal is for students to learn to handle scientific equipment like using a measuring angles, temperature, etc.
2. **Concept-based practical work:** learning new concepts like exploring surface area of amboïde, volume, etc.
3. **Inquiry-based practical work:** learning process skills. Examples of process skills are the following: defining the problem and good research question(s), installing an experimental setup, observing, measuring, processing data in tables and graphs, identifying conclusions, defining limitations of the experiment etc.

Note:

- To learn the new concept by practical work, the lesson should start with the practical work, and the theory can be explained afterwards (explore – explain). Starting by teaching the theory and then doing the practical work to prove what they have learned is demotivating and offers little added values for student learning.
- Try to avoid complex arrangements or procedures. Use simple equipment or handling skills to make it not too complicated and keep the focus on learning the new concept.
- The practical activity should be useful for all learners and not only for aspiring scientists. Try to link the practical work as much as possible with their daily life and preconceptions.

d. Organising practical activity**i) Methods of organizing a practical activity**

There are 3 methods of organizing a practical activity:

- **Each group does the same practical activity at the same time**

All learners can follow the logical sequence of the experiments, but this implies that a lot of materials needed. The best group size is 3, as all learners will be involved. With bigger groups, you can ask to do the experiment twice, where learners change roles.

- **Practical activities are divided among groups with group rotation**

Each group does the assigned experiment and moves to the next experiment upon a signal by the teacher. At the end of the lesson, each group has done every experiment. This method saves materials but is not perfect when experiments are ordered in a logical way. whereby next experiment depends on the results of previous experiments. In some cases, the conclusion of an experiment provides the research question for the next experiment. In that case, this method is not very suitable.

Before starting the lesson, the materials for each experiment should be placed in the different places where the groups will work. Also, the required time for each experiment should be the same. Use a timer to show learners the time left for each experiment. Provide an extra exercise for fast groups.

- **All practical activities are divided among groups without group rotation**

Each group does only one or two experiments. The other experiments are done by other groups. Afterward, the results are brought together and discussed with the whole class. This saves time and materials, but it means that each learner does only one experiment and 'listens' to the other experiments' description.

The method is suitable for experiments that are optional or like each other. It is not a good method for experiments that all learners need to master.

ii) Preparation of a practical work

When preparing a practical work, do the following:

- Have a look at the available materials at school and make a list of what you can use and what you need to improvise.
- Determine the required quantities by determining the method to apply (see above).
- Collect all materials for the experiments in one place. If learners' group is small, they can come to get the materials on that spot, but it is better for each group to prepare a set of materials and place it on their desk.
- Test all experiments and measure the required time for each step before the experiment.
- Prepare a nice but educational extra task for learners who complete their task before the end of allocated time.
- Write on the blackboard how groups of learners are formed.

iii) Preparation of a lesson for practical work

In the lesson plan of a lesson with practical work, there should be the following phases:

1. The introduction of the practical work or the 'excite' phase consists of formulation of a key question, discrepant event, or a small conversation to motivate learners and make connections with daily life and learners' prior knowledge.
2. The discussion of safety rules for the practical work. For example,
 - Learners must work at the assigned place.
 - Only the material needed for the experiment should be on the table.
3. Set the practical work instructions: how groups are formed, where they get the materials, special treatment of materials (if relevant), what they must write down, etc.
4. Set how to conduct a practical work:
 - Learners do the experiments, while the teacher coaches by asking questions (Explore phase).
 - The practical work should preferably be processed immediately with an explain phase. If not, this should happen in the next lesson.
5. Set how to conclude the lesson of a practical work:
 - Learners refer to instructions and conduct the experiment,
 - Learners record and interpret recorded data,

- Cleaning the workspace after the practical work (by the learners as much as possible).

e. Role and responsibilities of teacher and learners during practical activity

The roles and responsibilities of teacher during a practical activity

Before conducting an experiment, the teacher will do the following:

- Decide how to incorporate experiments into class content best,
- Prepare in advance materials needed in the experiment,
- Prepare protocol for the experiment,
- Perform in advance the experiment to ensure that everything works as expected,
- Designate an appropriate amount of time for the experiment - some experiments might be adapted to take more than one class period, while others may be adapted to take only a few minutes.
- Match the experiment to the class level, course atmosphere, and your students' personalities and learning styles.
- Verify lab equipment before practical work.
- Provide the working sheet and give instructions to learners during practical work.

During practical work, the teacher's role is to coach instead of helping with advice or questions. It is better to answer a learner's question with another question than to immediately give the answer or advice. The additional question should help learners to find the answer themselves.

- Prepare some questions for each practical work, no matter what the type is.
- Try and start the practical work: start with a discrepant event or questions that help define the problem or questions that link the practical work with students' daily life or their initial conceptions about the topic.
- Use coaching questions during the practical work: 'Why do you do this?', 'What is a control tube?', 'What is the purpose of the experiment?', 'How do you call this product?', 'What are your results?' etc.
- Use some questions to end the practical work: 'What was the meaning of the experiment?', 'What did we learn?', 'What do we know now that we didn't know at the start?', 'What surprised you?' etc.
- Announce the end of the practical work 10 minutes before giving learners enough time to finish their work and clean their space.

The Role of a lab technician during a lesson with practical activities

In schools having laboratory technicians, they assist the science and mathematics teachers in the following tasks:

- Maintaining, calibrating, cleaning, and testing the sterility of the equipment,
- Collecting, preparing and/or testing samples,
- Demonstrating procedures.

The learners' responsibilities in the practical work

During the experiment, both learners have different activities to do; the table below summarizes them. General learner's activities are:

- Experiment and obtain data themselves,
- Record data using the equipment provided by the teacher,
- Analyse the data often this involves graphing it to produce the related graph,
- Interpret the obtained results and deduce the theory behind the concept under the experimentation,
- Discuss the error in the experiment and suggest improvements,
- Cleaning and arranging material after the experiment.

f. Safety rules and precautions during practical activity

Safety rules and precautions during a practical activity

Regardless of the type of practical work you are in, there are general rules enforced as safety precautions. Each lab member must learn and adhere to the rules and guidelines set, to minimize the risks of harm that may happen to them within the working environment. Please make sure you are familiar with the safety precautions, hazard warnings, and procedures of the experiment you perform on a given day before you start any work. Experiments should not be performed without an instructor in attendance and must not be left unattended while in progress.

A. Hygiene plan

Mathematics experiments can be done in the classroom, outside, in the laboratory or in the math corner.

This place is a shared workspace, and everyone has the responsibility to ensure that it is organized, clean, well-maintained, and free of contamination that might interfere with the lab members' work or safety.

For waste disposal, all used materials must be discarded in designated containers. Keep the container closed when not in use. When in doubt, check with your instructor.

B. Hazard warning symbols

To maintain a safe workplace and avoid accidents, lab safety symbols and signs need to be posted throughout the workplace.

The hazard class will determine how similar materials should be stored and handled and what special equipment and procedures are needed to use them safely.

The annex 1 shows hazard symbols found in the working place and the corresponding explanations.

C. Safety rules

Safety is the number one priority in any working place. All students are required to know and comply with good working place practices and safety norms; otherwise, they will be asked to leave the working place. Make sure you understand all the safety precautions before starting your experiments, and you are requested to help your learners to understand too.

The following are some general guidelines that should always be followed:

- **Lab coat**

While working in the lab, everyone must always wear a lab coat (Figure 1) to prevent incidental and unexpected exposures to the skin and clothing. The primary purpose of a lab coat is to protect against splashes and spills.



The lab coat must be wrist-fitted and must always keep buttoned.

A lab coat should be non-flammable and should be easily removed.

- **Breathing Masks**

Respirators are designed to prevent contamination from volatile compounds that may enter in your body through the respiratory system. “Half mask” respirators (Figure3) cover just the nose and mouth; “full face” respirators cover the entire face, and “hood” or “helmet” style respirators cover the entire head.



The breathing mask safety sign lets you know that you are working in an area with potentially contaminated air.

- **Footwear**

Shoes that cover entirely the toes, heel, and top of the foot provide the best general protection (Figure 1.5). Closed shoes must always be worn while in the laboratory, regardless of the experiment or curricular activity. Shoes must fully cover your feet up to the ankles, and no skin should be shown.



Socks do not constitute a cover replacement for shoes. Sandals, backless and open shoes are unacceptable.

- **Gloves**

When handling any hazards that can enter the body through the skin, it is important to wear the proper protective gloves.



- **Hair dressing**

If hair is long, it must be tied back. It is good to report all accidents including minor incidents to your instructor immediately.

- **Eat and drink**

Never drink, eat, taste, or smell anything in the experiment place unless you are allowed by the lab instructor.

- **Hot objects**

Never hold very hot objects with your bare hands.



Always hold them with a test tube holder, tongs, or a piece of cloth or paper.

g. Guidance on the Management of Math kit items: Storage Management, Repairing and Disposal

Keeping and cleaning up

Working spaces must always be kept neat and cleaned up before leaving. Equipment must be returned to its proper place. Keep backpacks or bags off the floor as they represent a tripping hazard.

Management of materials

A laboratory or math corner is a place where basic experimental skills are learned only by performing a set of prescribed experiments. Safety procedures usually involve waste disposal procedures. In the laboratory, materials should be stored in their original containers, and cabinets should be suitably ventilated. It is important to notify students that some materials made in glass cannot be stored in containers on the floor. Sharp and pointed tools should be stored properly.

Students should always behave maturely and responsibly in the laboratory or wherever chemicals are stored or handled.

- **Hot equipment and glassware handling**

All glassware must be handled carefully and stored in its appropriate place after use. When working in a lab, do never leave a hot plate unattended while it is turned on. It is recommended to handle hot equipment with safety gloves and other appropriate aids but never with bare hands. You must ensure that hands, hair, and clothing are kept away from the flame or heating area and turn heating devices off when they are not in use in the laboratories.

- **Waste disposal considerations**

Waste disposal is a normal part of any science or Math laboratory. As teachers or students perform demonstrations or experiments, chemical waste is generated.

These wastes should be collected in appropriate containers and disposed of according to local, and national regulations. All schools should have a person with the responsibility of being familiar with this waste disposal. In order to minimize the amount of waste generated and handle it safely, there are several steps to consider.

Sinks with water taps for washing purposes and liquid waste disposal are usually provided on the working table. It is essential to clean the sink regularly. Notice that you should never put broken glass or ceramics in a regular waste container. Use a dustpan, a brush, and heavy gloves to carefully pick-up broken pieces, and dispose of them in a container specifically provided for this purpose.

- **Equipment Maintenance**

Maintenance consists of preventative care and corrective repair. Both approaches should be used to keep equipment in working order. Records of all maintenance, service, repairs, and histories of any damage, malfunction, or equipment modification must be maintained in the equipment logs. The record must describe hardware and software changes and/or updates and show the dates when these occurred. Each school must have an inventory of Math kit items that should be updated at least once a year.

- i. Student's Experiment Work Sheet**

There should be a sheet to guide students about how they will conduct the experiment, materials to be used, procedures to be followed and the way of recording data. The following is a structure of the student experiment worksheet. It can be prepared by teacher or be availed from the other level.

1. Date
2. Name of student/group
3. The title of experiment
4. Type of experiment (concept, equipment and inquiry based)
5. Objective(s) of the experiment
6. Key question(s)
7. Materials (equipment/instrument, resources, etc...)
8. Procedures & Steps of experiment
9. Schematic reference if required.

10. Table of data recording and presentation

| Number of tests | Variables | Results | Comments/ Observations |
|-----------------|-----------|---------|---------------------------|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| Etc | | | |

11. Reflective questions and answers

Question 1

Question 2

Question 3

12. Answer for the key questions or conclusion.

j. Report Template for students

After conducting a an experiment, students should write a report about their findings and the conclusion they took.

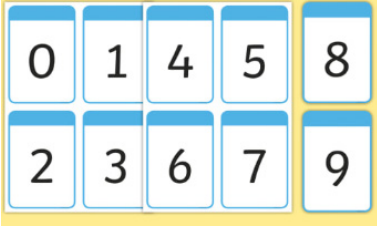


The report to be made depends on the level of students. The report done by primary school learners is not the same as the one to be made by secondary school learners.

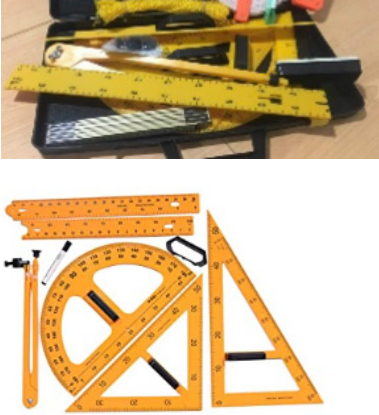


The following is a structure of the report to be made by a group of secondary school learners.



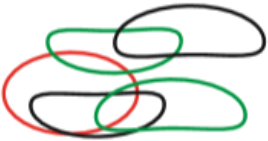

1. Introduction (details related to the experiment: Students identification, date, year, topic area, unit title and lesson).
2. The title of experiment.
3. Type of experiment (concept, equipment and inquiry based)
4. Objective(s) of the experiment.
5. Key question(s)
6. Materials (equipment/instrument, resources, etc...)
7. Procedures & Steps of experiment
8. Schematic reference if required.
9. Data recording
10. Data analysis and presentation (Plots, tables, pictures, graphs)
11. Interpretation/discussion of the results, student alternative ideas from observation.
12. Theory or Main ideas concept, formulas, and application).
13. Conclusion (answer reflective questions and the key question).

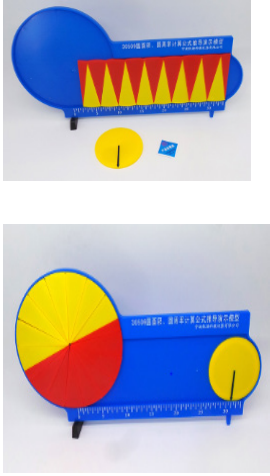
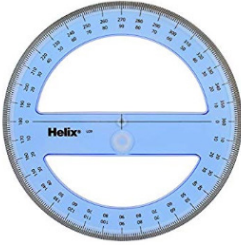
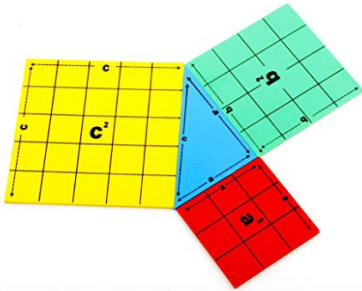
As a conclusion, there are safety rules and precautions to consider before, during and at the end of an experiment. We hope teachers are inspired to conduct experiments and practical activities in a conducive Competence Based Curriculum way.

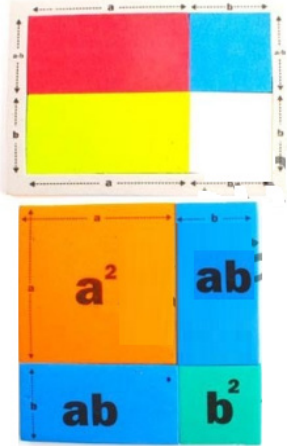
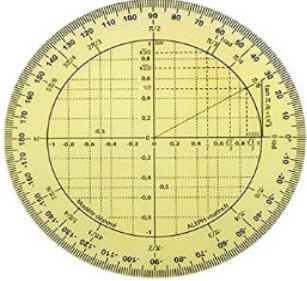
PART 2: LIST OF MATHEMATICS KIT ITEMS DISTRIBUTED IN SCHOOLS

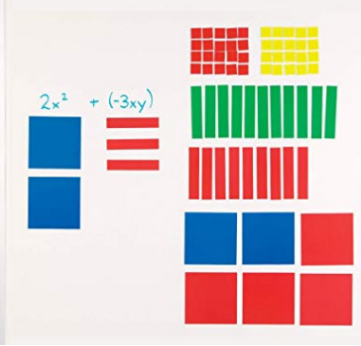


| # | Item and description | Picture | Description |
|---|---|---|--|
| 1 | <p>Laminated number cards</p> <p><i>Use: Used in game for composition, sorting, factorization of numbers, etc.</i></p> |  | <p>A pack of laminated cards numbered from 0 to 9 (9 cards from an A4 paper).</p> |
| 2 | <p>Circle set fraction</p> <p><i>Use: Used for exploring "Area of Circle" and activities related to "Fractions" and area of a circle</i></p> |  | <p>7 Blue (or any other color) colored circular plastic having 3mm thickness and diameter 160 mm. divided into 4, 6, 8, 12, 16 and 32 equal sectors.</p> <p><i>Each piece is magnetic.</i></p> |
| 3 | <p>Clock</p> <p><i>Use: To learn to tell the time according to the 24 hours international convention.</i></p> |  | <p>1 plastic teaching clock</p> |


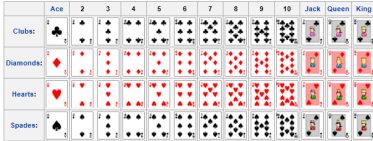

| | | | |
|-----------------|--|--|--|
| <p>4</p> | <p>Mathematical set for teachers: Full circle protractor, meter rule, compass, tape measure, T-square, rope, decameter.</p> |  | <p>Wooden or plastic</p> |
| <p>5</p> | <p>Mathematical set for students: 2 Metal Study Compasses, 2 T-squares, Ruler, Protractor, Pencil for Compass, Pencil Sharpener, Eraser, Lead Refill.</p> |  | <p>Geometry 10 Piece Set,</p> |
| <p>6</p> | <p>Fest night Stainless Steel 180 Degree Protractor</p> |  | <p>Angle Finder Both Arms Stainless Steel Protractor with 0-180 Degrees, Angle 10 inch, 250mm, 30cm Scale Angle Finder Ruler. Smooth surface, convenient to use, easy to read. 0-180 degree arbitrary rotation. Adjustable screw design, easy operation for fixed reading.</p> |

| | | | |
|----|---|---|---|
| 7 | <p>Basic geometric solids</p> |  | <p>6 pieces of wooden solids Includes cube, cylinder, sphere, cone, triangular prism, pyramid .</p> <p><i>Use: To demonstrate geometry solids (3D).</i></p> |
| 8 | <p>Geoboard</p> <p><i>Geoboard is used to represent planar shapes/ figures and also to find the approximate areas as well as to learn different geometric figures using a rubber band.</i></p> |  | <p>1 geographic board of 33.5 cm × 53.5 cm. It is printed with 187 grids 3 cm × 3 cm each in alternated colors. Copper pins are nailed on each crossing point of the grids.</p> |
| 9 | <p>Rubber bands</p> <p><i>for use with geoboard.</i></p> |  | <p>120 rubber bands in 6 colors come with the boards.</p> |
| 10 | <p>Transparent geometric 3D-shapes plus their corresponding fold-up nets: cylinder, square pyramid, cube, rectangular prism, cone, hexagonal prism, triangular pyramid, and triangular prism.</p> <p><i>Use: Used to make solid shape.</i></p> |  | <ul style="list-style-type: none"> • Transparent geometric shapes plus their corresponding fold-up net inserts. • 16-piece set (8 transparent and 8 folding shapes) |

| | | | |
|------------------|---|---|--|
| <p>11</p> | <p>Circle-Area and Diameter Demonstrator</p> |  | <p>1 plastic demonstrator board of 48 cm × 25 cm. It consists of 17 sectors: 15 sectors that are equal to 1/16 of the cylinder volume; and 2 sectors that are equal to 1/32 of the cylinder volume. Use: To learn how to measure the area and diameter of circles.</p> |
| <p>12</p> | <p>Full Protractor</p> |  | <p>Helix Professional 360 Degree Protractor 15cm As per sample</p> |
| <p>13</p> | <p>Cut Outs for Pythagoras Theorem.</p> |  | <p>1 plastic right angled triangle. Measure: 3" x 4" x 5" & 3 different size square equal to sides of triangle.</p> |

| | | | |
|------------------|--|---|---|
| <p>14</p> | <p>Cut outs for algebraic Identities.</p> |  | <p>Made up of 3mm thick wooden colored cardboard and includes</p> <p>(1) Square cut-out of side 76 mm</p> <p>(2) 3 cut-outs obtained from another square of side 76mm out of which one is a square of side 38mm and the remaining two are trapezium of dim.38 x76 mm^2</p> <p>(3) Square cut-out s of side 80 mm and 45mm</p> <p>(4) Rectangular cut-out of dimension 80x45mm^2</p> |
| <p>15</p> | <p>Circular Trigonometric Protractor</p> |  | <p>CIRCULAR Protractor ROBUST: shatterproof and scratch resistant translucent plastic.</p> <p>Use: direct reading of the angles in degrees and radians and cosine and sine near 0.05.</p> |

| | | | |
|------------------|--|---|---|
| <p>16</p> | <p>Algebraic tiles</p> <p>a) $x^2, x, 1$ b) $-x^2, -x, -1$</p> |  | <p>Made up of plastic cardboard in different sizes:</p> <p>40 (20red+20 blue) squares of side 10mm known as unit tiles.</p> <p>20 (10 red +10 blue) rectangles of $50 \times 10\text{mm}^2$ dimension known as x or -x tiles.</p> <p>10 (5 red + 5 blue) squares of side 50mm known as x^2 or $-x^2$ tiles, etc.</p> |
| <p>17</p> | <p>Cubic dice</p> <p>From 1 sided to 6 sided.</p> |  | <p>6 plastic dices with different edges and different shapes: 8mm, 12mm, 16mm, 19mm and 25mm.</p> |
| <p>18</p> | <p>Counters: (20)</p> <p><i>Use: Used in activity "Addition and Subtraction of Integers".</i></p> |  | <p>A set of 20 Plastic pieces or laminated transparent counters whose one side is blue and other side is red.</p> |

| | | | |
|----|---|---|---|
| 19 | Scientific Calculator |  | Casio Fx-991es Plus Scientific Calculator |
| 20 | Playing cards to be used in probability |  | A set of 52 playing cards |
| 21 | The container: a box in metal to contain all these materials per kit. |  | A container in metal which can contain or these materials |

PART 3: PRACTICAL ACTIVITIES OR EXPERIMENTS PER GRADE

PRACTICAL ACTIVITY 1: Representation of a problem using a Venn diagram involving two sets

a) Rationale:

This activity is conducted when teaching the lesson on set operations: intersection and union of 2 sets. It is taught in unit 1 of S1.

In real life, a set is a group or collection of objects or numbers, considered as an entity unto itself. Each object or number in a set is called a member or element of the set. Examples include the set of all computers in the world, the set of all apples on a tree, and the set of all irrational numbers between 0 and 1. Sets are usually used to represent, collect, and study similar data. Data is a very important aspect in today's life.

b) Objectives:

- To discover that intersection is the set of common members in two groups.
- To clearly illustrate union of sets, intersection of sets and complement of set.

c) Hazard notification

Students must care of the use of razor blade and pair of scissors as they can cut them.

Since bottles are none degradable materials, dump them in prepared waste disposal after the experiment. Otherwise, they are harmful to the environment.

d) Required materials

Pair of scissor or (razor blade), bottles of Inyange, two strings with two different colours (red and black), blue and white sheets of paper.

e) Procedure and steps

Step 1: Cut the white sheet of paper in white labels used as indicators which show product verified by Rwanda Standards Board (RSB).

Step 2: Cut blue sheet of paper in blue labels used as indicators which show product verified by Rwanda Utilities Regulatory Authority (RURA).

Step3: Put white labels on 6 bottles.

Step 4: Put blue labels on 4 bottles including 2 bottles that also have white labels.

Step 5: Collect the bottles verified by RSB in their own string with black color.



Step 6: Collect again the bottles verified by RURA in their own string with red color.

Step 7: Identify bottles verified by RURA and RSB.

- If A is the set of bottles verified by RSB, how many elements does it have?
- If B is the set of bottles verified by RURA, how many elements does it have?
- How many bottles were verified by RSB?
- How many bottles were verified only by RURA?
- How many bottles were verified only by RSB only?
- If $A \cap B$ means the intersection of the set A and set B means the common elements of the two sets A and B . How many bottles are in $A \cap B$? Use RSB and RURA to explain $A \cap B$.
- Find the number of bottles verified by RURA or RSB. Can we say that their number is the sum of bottles verified by RURA and bottles verified by RSB? Explain your answer.

f) Data recording:

Number of bottles verified by RURA=4

Number of bottles verified by RSB =6

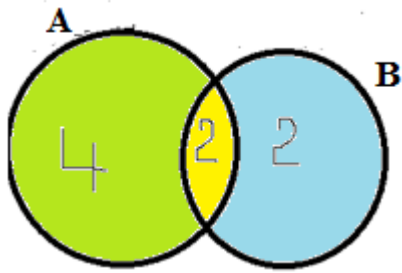
Number of bottles verified by RURA only =2

Number of bottles verified by RSB and RURA=2

Number of bottles verified by RSB or RURA =8

g) Interpretation of results and conclusion

In Mathematics, we can write A as the set of bottles verified by RSB and B the set of bottles verified by RURA. In the Venn diagram below, there are the number of bottles but we should have bottle 1, bottle 2, etc.



Express the following in words:

$$n(A) = 6$$

$$n(B) = 4$$

$$n(A \cap B) = 2$$

$$n(A \cup B) = 8 = n(A) + n(B) - n(A \cap B)$$

$$n(A') = 2$$

$$n(B') = 4$$

h) Conclusion

The above explanations give us the following:

$n(B)$: Number of bottles verified by RURA=4=

$n(A)$: Number of bottles verified by RSB

$n(B')$: Number of bottles verified by RSB only = 4

$n(A')$: Number of bottles verified by RURA only =2.

$n(A \cap B)$: Number of bottles verified by both RSB and RURA=2

$n(A \cup B)$: Number of bottles verified by RSB or RURA =8

PRACTICAL ACTIVITY 2: Representing set operations in a Venn diagram involving 3 sets

a) Rationale:

This activity can be done when teaching Venn diagrams and set operations. It is taught in unit 1. In real life, a set is a group or collection of objects or numbers, considered as an entity unto itself. Each object or number in a set is called a member or element of the set. Examples include the set of all computers in the world, the set of all apples on a tree, and the set of all irrational numbers between 0 and 1. Sets are usually used to represent, collect, and study similar data. Data is a very important aspect in today's life.

b) Objective:

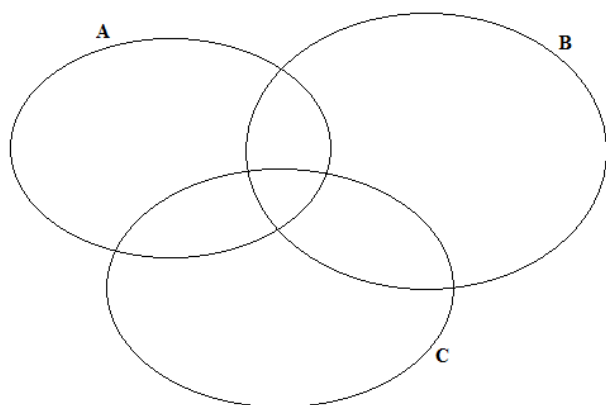
Use Venn diagram involving 3 sets to represent mathematical real-life situations.

c) Required materials:

- Number cards with the numbers: 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10.
- Manila papers, markers, protectors, compass, bucket lid/covers, bicycle wheel rims, pencils, chalks, and pens.

Note: If manila papers are not available, use a piece of chalk to draw sets on a playground.

d) Activity set up (illustration)



e) Procedures

Step 1: Draw three intersecting Venn diagrams on either manila papers, a playground, an open box, piece of rice sack, or piece of plywood, and name them A, B, C;

Such that:

- A:** set of positive odd counting numbers less than 10.
- B:** set of positive even counting numbers less than 10.
- C:** set of positive prime counting numbers less than 10.

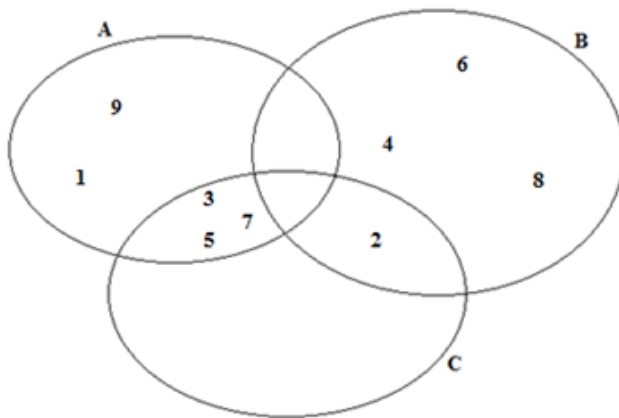
Step 2: In groups, take the number cards and discuss where to hang/place its number on a figure/Venn diagram.

Step 3: Study the Venn Diagram and answer to the following questions and place each number card in the appropriate place on the Venn diagram:

- Which numbers are at the same time odd and prime numbers? What does this represent Mathematically?
- which numbers are even and prime at the same time? what does this represent in mathematics?
- Which numbers are even or prime numbers? What does this represent mathematically?
- What numbers are even or prime or odd?
- Which numbers are even only? How can their set be represented mathematically?

f) Results, their interpretation and Conclusion

We see that the number cards are hanged as follow:



- The numbers 3,5,7 are in both Sets A and C. These numbers are at the same time odd and prime numbers. In mathematics, they form the intersection of the set A and the set C. Therefore, $A \cap C = \{3,5,7\}$
- The number {2} belongs to set B and C. this number is at the same time even and prime number. Therefore, $B \cap C = \{2\}$
- The numbers 2,3,4,5,6,7,8 are even or prime numbers. In Mathematics, they form union of set B and C. Therefore, $B \cup C = \{2,3,4,5,6,7,8\}$
- The numbers 1,2,3,4,5,6,7,8,9 are odd or prime or even numbers. Mathematically, they form the Union of sets A, B and C. Therefore, $A \cup B \cup C = \{1,2,3,4,5,6,7,8,9\}$

- The numbers 4,6,8, are only even numbers. Mathematically is called simple difference of set B and C. Therefore $B - C = \{4, 6, 8\}$ or $B - (A \cup C) = \{4, 6, 8\}$

Note: Students may misplace a number card on Venn diagram, move around to each group to verify and provide more questions leading to the good understanding.

g) Guidance on the evaluation

Give students another practical activity involving the representation of 3 sets on a Venn diagram:

Ask every student to take a sheet of paper and write the subject he/she like among Mathematics (M), Physics (P) or Chemistry (C). Tell them that each one can write one subject only, two subjects or 3 subjects at the same time.

Tell them to go to the playground, draw a big Venn diagram involving the 3 sets:

P: Number of students who like physics,

C: Number of students who like chemistry,

M: Number of students who like Mathematics.

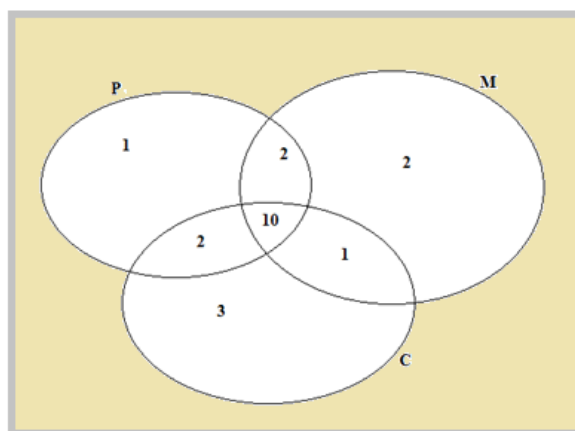
Then, ask each one to stand in the appropriate position depending on the subjects he/she likes.

Example of expected answer

In a class of 21 students, teacher received the following sheets:

- 2 sheets: Physics only
- 2 sheets: Physics and Mathematics,
- 10 sheets: Physics, Mathematics and Chemistry.
- 1 sheet: Mathematics and chemistry
- 3 sheets: Chemistry only,
- 2 sheets: Physics and chemistry,
- 2 sheets: mathematics only.

Students got stand in the drawn Venn diagram as follows:



Ask some questions to interpret and draw conclusion on the situation.

PRACTICAL ACTIVITY 3: Exploring relations between sets, defining domain and range, and creating mappings

a) Rationale:

This activity can be done when teaching relations (mappings). It is taught in unit 1 of S1. Relation and Function in real life give us the link between any two entities. In our daily life, we come across many patterns and links that characterize relations such as a relation of a father and a son, brother, and sister, etc. In mathematics also, we come across many relations between numbers such as a number x is less than y , line l is parallel to line m , etc. Relation and function map elements of one set (domain) to the elements of another set (codomain).

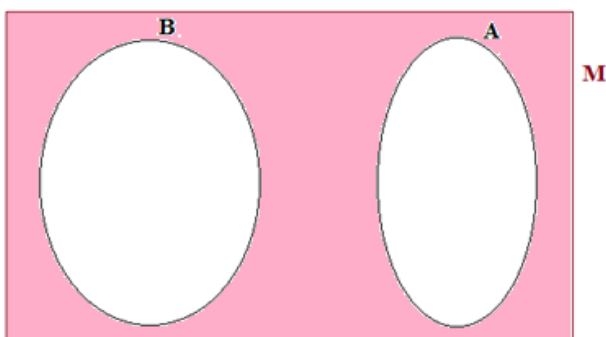
b) Objective

Use mappings to represent relations in real life situations.

c) Required materials:

- Object cards (picture of objects): leaf of beans, meat, and maize.
- Animal cards (picture of animals): hen, cow, goat, and dog.

d) Illustration of the activity



e) Procedures

Step 1: From the following set $M = \{\text{leaf of bean leaves, meat, maize grains, hen, cow, goat, dog}\}$;

Form a set A of objects that animals eat and a set B of animals.

Step 2: Observe the set A , B and M and complete:

The set A is a part of the set....., Mathematically we write: $B \dots M$.

The set B is a part of the set, Mathematically we write $A \dots M$

The set A and the set B form the set M . Mathematically we write:

$M = A \dots B$.

Step 3: Consider that there is a relation “.....*usually eats*.....” that associates the element of the set A and the element of the set B , and do the following:

- Use a manila paper and draw a set A and the set B separately.
- Use object cards and animal cards and place them in the set A , B and M ,

iii) Use sticks to join each card from B by another card from A respecting the relation R: “.....**usually eats**.....”

iv) Complete the set of ordered pairs of the relation R.

v) Discuss if the relation R is a mapping or not.

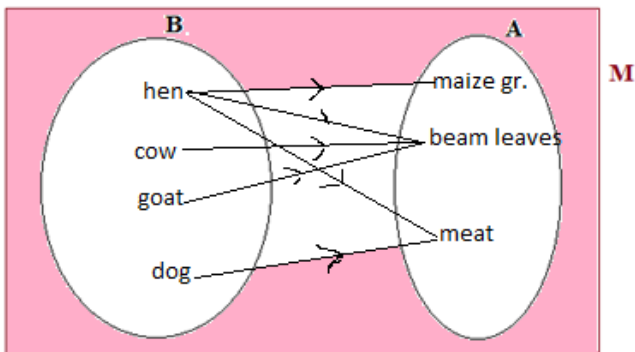
If it is a mapping, What kind of mapping is it?

Give the domain and the range of the relation R.

vi) Can you conclude that there is a relationship between objects, things or people we see in our environment?

f) Results and conclusion

From the arrow diagram below, the questions i), ii), iii) are answered.



iv) The set of ordered pairs of the relation R: {(hen, leaf of beans), (hen, maize), (hen, meat), (cow, leaf of beans), (goat, leaf of beans), (dog, meat)}.

v) Since each element of the set B is associated to the element of the set A, the relation R is a mapping defined from the set B to the set A.

Set B is the domain while set A is the Range.

vi) In our real life, we can find the relation between objects themselves, animals themselves, people themselves, people and objects, people, and animals, etc.

g) Information for teacher:

A mapping is a mathematical relation such that each element of a given set (the domain) is associated with an element of another set (the range)

h) Guidance on the evaluation:

Ask students to carry out the following activity.

1. You are provided with the following to form the domain set: cat, dog, buffalo, lion, hen, duck, cow.
2. You are also given the following to form the range: carnivorous, herbivorous, domestic, wild, bird, omnivorous.
3. On a sheet of paper, show the above mapping and explain it.
4. Are there elements in the domain set that map onto several elements in the range set?

PRACTICAL ACTIVITY 4: Forming and measuring angles

a) Rationale:

This activity is done when teaching the types of angles. It is taught in unit 6 of S1.

In real life, engineers use angle measurements to construct buildings, bridges, houses, monuments, etc. Carpenters use angle measuring devices such as protractors, to make furniture like chairs, tables, beds, etc. The angle can be seen in the wall clocks of our homes, made by hands of clocks.

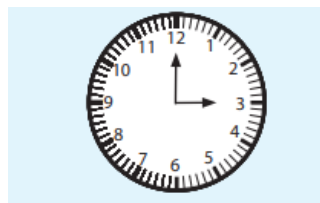
b) Objective:

To form practically the angles and measure their degrees.

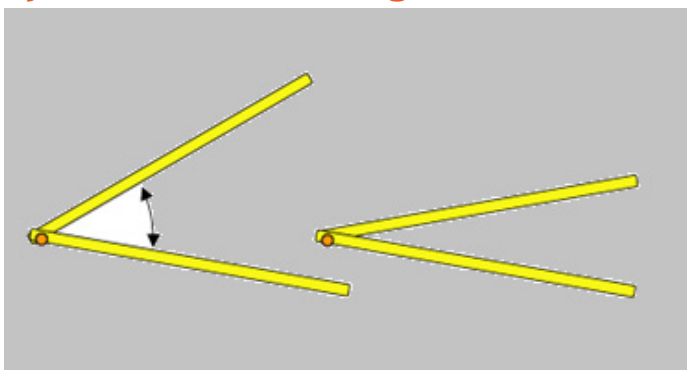
c) Required materials

To conduct this activity, use the following materials:

- Protractor
- A4 Plain Papers
- Wall clock
- Set square
- Compass.
- Fest night stainless steel degree protractor.
- Pencils.
- Rectangular base objects, like box, books, ruler, etc

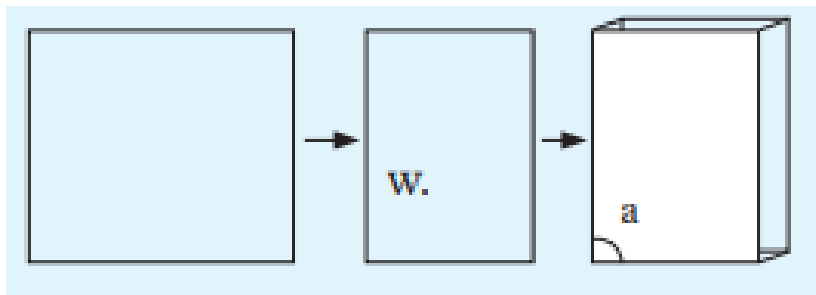


d) Illustration of an angle



e) Procedures

Step1. Fold the paper into two equal halves. Fold it again into two equal halves.



- What is the name of angle shown by the letter **a** ?
- Use the protractor to measure the angle formed by two edges?

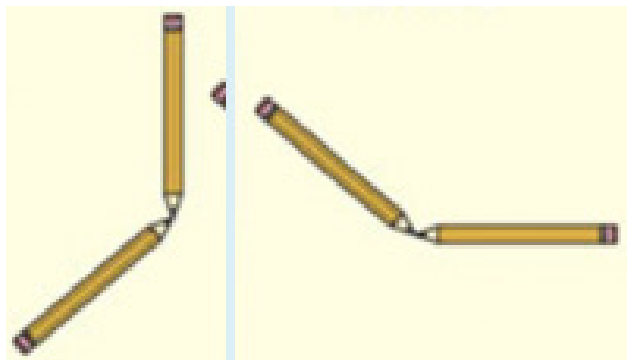
Step 2. Take a fest night stainless steel degree protractor and form an angle whose value is less than a right angle. What value is it?

Step 3. Take a fest night stainless steel degree protractor and form an angle whose value is greater than a right angle, what value is it?

Step 4. Obtain and observe a wall clock or a clock face. What type of angles formed between the hour hand and the minute hand?

- At 3.00 o'clock?
- At clock at 2.00 o'clock?
- At clock at 4.00 o'clock?
- At clock at 6.00 o'clock?
- At clock at 8.00 o'clock?

Step 5: Take pencils, form more different angles, and say their types.



f) Interpretation of results and Conclusion

- The angle formed by the square corner of a folded paper is called a **right angle**.
- At 3.00 o'clock, the type of angle formed between the minute hand and the hour hand is a **right angle**.
- At 4.00 o'clock, the type of angle formed between the minute hand and the hour hand is an **obtuse angle**.

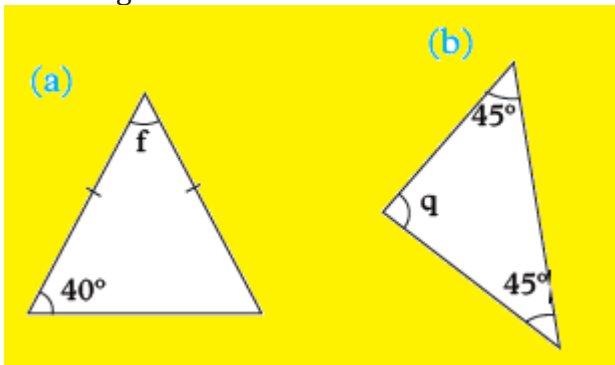
- iv. At 2.00 o'clock, the type of angle formed between the minute hand and the hour hand is **an acute angle**.
- v. At 6.00 o'clock, the type of angle formed between the minute hand and the hour hand is **a straight angle**.
- vi. At 8.00 o'clock, the type of angle formed between the minute hand and the hour hand is **reflex angle**.

g) Information for teacher

- i. The right angle is equivalent to 90° on a protractor.
- ii. The obtuse angle is greater than 90° and less than 120° .
- iii. The acute angle is greater than 0° but less than 90° .
- iv. The straight angle is measure 180°
- v. The reflex angle is greater than 180° and less than 360°

h) Guidance on the evaluation

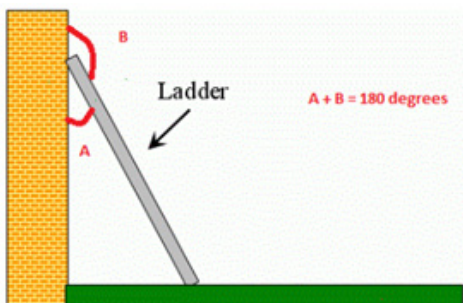
Ask students to state the type of angles marked with letters in each of the following:



PRACTICAL ACTIVITY 5: Verifying the relation among the angles formed by a transversal of parallel lines

a) Rationale:

This activity can be done when teaching the angles on a parallel line. It is taught in unit 6 of S1. In our daily routine, we face several situations where the concept of Linear Pair of angles is applied. Even if we don't notice, there are many things around us which form the part of Linear Pair.



A ladder placed against the wall is a real-life example of Linear Pair. Here, the 'angle A' formed by placing the ladder against the wall is adjacent to 'angle B.' Both the angles formed have a common vertex and the sum equal to 180 degrees.

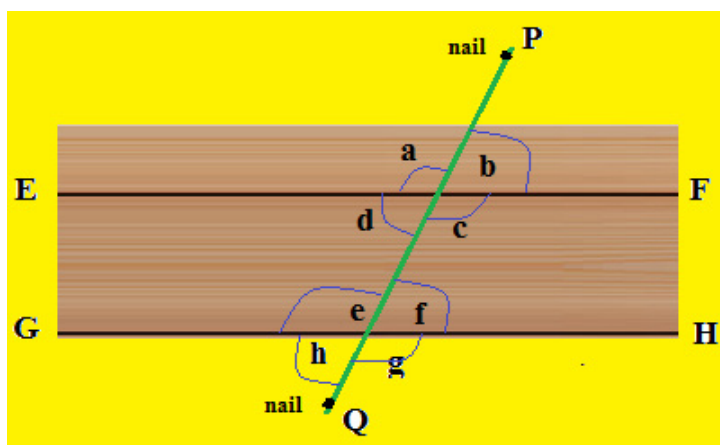
b) Objective

To verify angles formed by parallel lines and a transversal

c) Required materials

- Two well prepared timbers of different heights, nails, thread, ruler, protractor, pens, pencils, markers.

d) Activity set up (illustration)



e) Procedures

Step 1: Take two flat pieces of wood and arrange them on a table as above.

Step 2: With the help of nails fix the thread in such a way that it is transversal to the pieces of timber from point Q to point P as shown above.

Let EF be the line of junction of two timbers and let GH be the line of extremity for the second timber.

i) What is the relationship between lines EF and GH?

ii) What is the name of the line indicated by the thread from point Q to P?

Step 3: Use a protractor to measure the values of angles a, b, c, d formed at the intersecting point between the thread and the line EF and angles e, f, g, h formed at intersecting point between the thread and the line GH of the second timber. Hence, complete the table below

| Angle | a | b | c | d | e | f | g | h |
|-------|---|---|---|---|---|---|---|---|
| Value | | | | | | | | |

Step 4: Compare the size of angle a and e; b and f, c and g; d and h; what do you notice? What is the name of the angle pairs?

Step 5: Compare the size of the angle pairs d and f, e and c. What do you notice? What is the name of the angle pairs?

Step 6: Compare the size of the angle pairs a and c, e and g, d and b, f and h. What do you notice? What is the name of the pair of angles?

Step 7: Compare the size of the angle pairs a and b, c and d, e and f, g and h. What do you notice? What is the name of the angle pair?

Step 8: Compare the size of the angle pairs d and e, c and f. What do you notice? What is the name of the angle pair?

f) Results interpretation and Conclusion

From Step 4: Angles in pairs: a and e; b and f, c and g; d and h are corresponding angles and respectively equal.

From Step 5: Angles in pairs: d and f, e and c are alternate angles and respectively equal.

From Step 6: Angles in pairs: a and c, e and g, d and b, f and h are opposite angles and respectively equal.

From Step 7: Angles in pairs a and b, c and d, e and f, g and h are supplementary angles and add up to 180° .

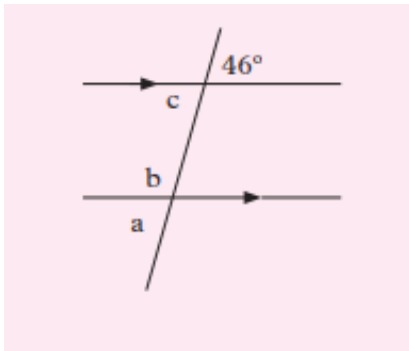
From Step 8: The angles in pairs: d and e, c and f are co-interior angles and add up to 180°

g) Information for teacher:

- i. A transversal line is a straight line which cuts through two lines on the same plane at distinct points.
- ii. The corresponding angles are equal.
- iii. Alternate angles are equal.
- iv. Co-interior angles add up to 180°
- v. Supplementary angles: their sum form an angle of 180 degrees (add up to 180°).
- vi. vii. Opposite angles are equal.

h) Guidance on the evaluation

Ask students to calculate the angles marked with letters in each of the following:



Expected answer:

$c = 46^\circ$ because they are vertically opposite angles.

$a = c = 46^\circ$ they are corresponding angles.

$b = 180^\circ - a$ because they are on a straight line/ supplementary angles.

$b = 180^\circ - 46^\circ$

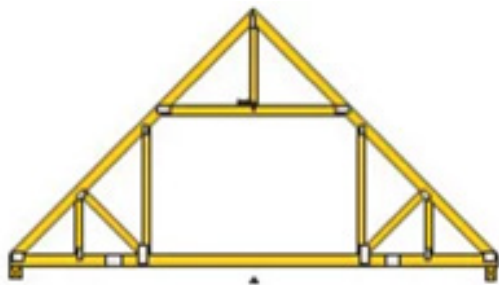
$b = 134^\circ$

PRACTICAL ACTIVITY/ EXPERIMENT 6: Exploring the angles of a triangle

a) Rationale:

This experiment is done when teaching the angles in a triangle. It is taught in unit 6 of S1.

There are many examples of triangles used in daily life. Looking at the side view of many houses, you will see the simple shape of a triangle above a rectangle.



If you see the house being built before the siding or bricks are put on, you can see the wooden studs for the walls of the rectangle and the rafters and ceiling beams that form the triangle. Similarly, if you look at an outdoor swing set from the side view, you will notice the two legs and the ground make an isosceles triangle shape. The designing of such triangular forms considers angles made by their sides.

b) Objective:

To verify practically that the sum of angles of a triangle is 180° .

c) Required material:

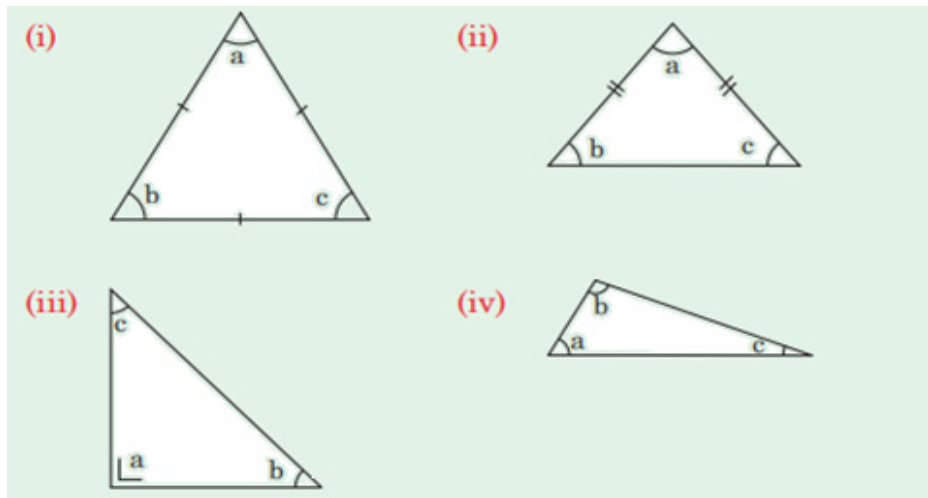
Sheets of paper.

Protractor

Set square

d) Procedures

Step 1: Cut papers into many triangles with different shapes and name their angles a, b, and c



Step 2: Use protractor to measure the angles a, b, c.

Step 3: In each case, find the sum of $a + b + c$.

| Angle | a | b | c | <u>$a+b+c$</u> |
|------------|---|---|---|---------------------------|
| Triangle 1 | | | | |
| Triangle 2 | | | | |
| Triangle 3 | | | | |

What do you note about the sum of angles in each triangle?

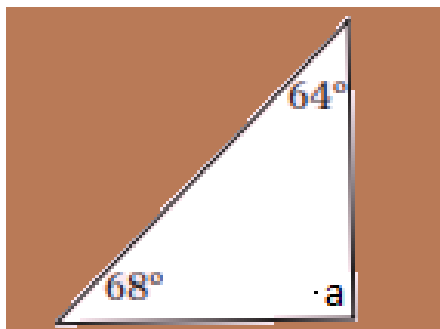
e) Interpretation of results and Conclusion

In each case, we find that interior angles of a triangle add up to 180° .

$$a + b + c = 180^\circ$$

f) Guidance on the evaluation.

In groups, Ask students to discuss the value of the angle marked by letter a on part of a timber with the following form:



Expected answer:

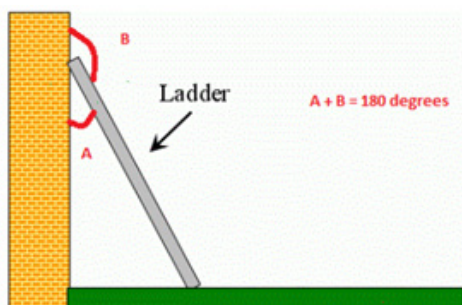
$$\text{Angle } a \text{ is } 180^\circ - (64^\circ + 68^\circ) = 48^\circ$$

PRACTICAL ACTIVITY/ EXPERIMENT 7: Forming a straight angle

a) Rationale:

This experiment is done when teaching the angles formed on a straight line. It is taught in unit 6 of S1.

In real life, we every day observe where two parts of an object for the angle of 180° , which makes is a straight angle. For example, a ladder placed against the wall is a real-life example of objects whose pair of angles form a straight angle.



Here, the 'angle A' formed by placing the ladder against the wall is adjacent to 'angle B.' Both the angles formed have a common vertex and the sum equal to 180 degrees.

b) Objective:

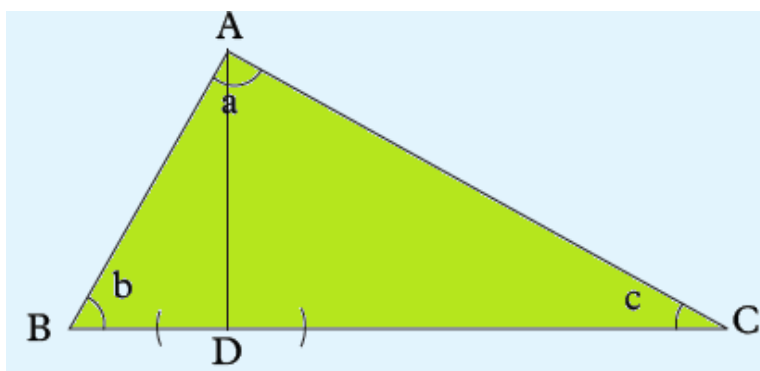
To verify that the sum of angles formed on a straight line is equal to 180°

c) Required materials

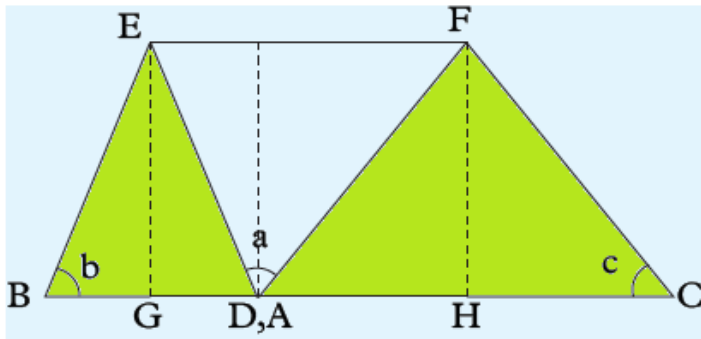
To conduct this activity, use the following materials:
Protractor, A4 Plain Papers, Set square, Compass

d) Procedures

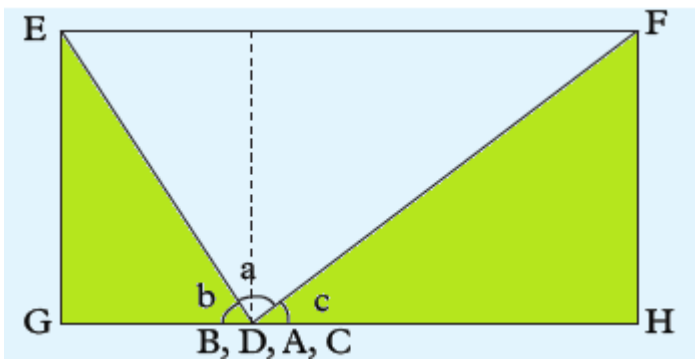
Step1. Form a triangle ABC with angles **a**, **b**, **c** and drop a perpendicular from point A to line BC at D. Cut out triangle ABC.



Step 2. Fold the paper and cut out in such a way that vertex A lies exactly on D, then label the fold line EF. What is the name of figure formed?



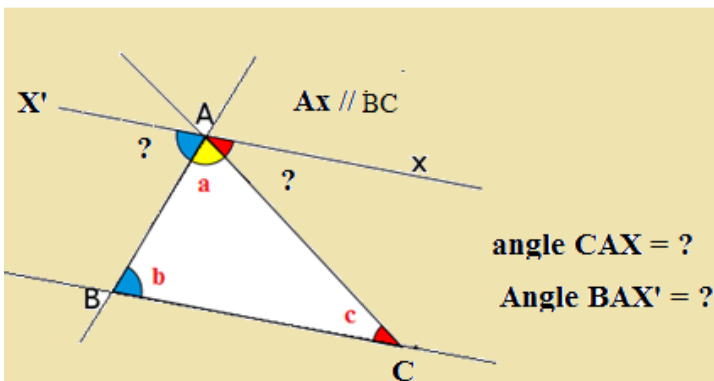
Step 3: Fold triangle BED along its vertical perpendicular such that B lies on D and fold triangle FDC along its perpendicular FH such that C lies on D.



- What figure did you obtain?
- On the line GH, measure angles a, b, c, and add them. What do you observe from the results?
- What can you say about the angles formed on the straight line in general?

Step 4: In the same case,

Consider the parallel lines $X'X$ and the line BC, transversal lines AB and AC and triangle ABC. Observe and compare the angles formed on the line AX and the angles of triangle ABC. Comment on your findings.



e) Interpretation of results and Conclusion

Basing on the findings:

From step 3:

i. Figures obtained include: triangle EGD, EDF and FHD; Rectangle GEFH
Angles a, b and c form a straight angle on the line GH and they add up to 180°

From step 4:

i. Angle CAX = the angle c and the angle BAX' = angle b.

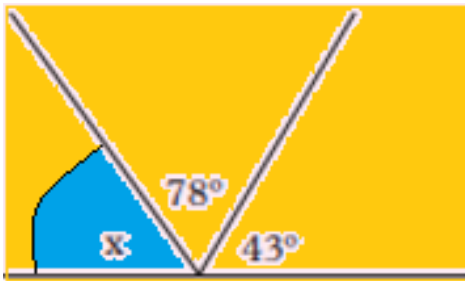
Therefore, the angle formed on the line X'X is $a + b + c = 180^\circ$.

f) Information for teachers

Angles formed on a straight line add up to 180° (make a straight angle of 180°).

g) Guidance on the evaluation.

Ask students to find the size of the angle marked with letter x on a part of a timber illustrated below:



Expected answer:

$$x + 78^\circ + 43^\circ = 180^\circ \text{ (angles on a straight line)}$$

$$x + 121^\circ = 180^\circ$$

$$x = 180^\circ - 121^\circ$$

$$x = 59^\circ$$

PRACTICAL ACTIVITY/ EXPERIMENT 8: Exploring Angles at the point

a) Rationale:

This experiment can be done when teaching the Angles at the point. It is taught in unit 6 of S1. In real life, we all know the minute hand of a clock face, it is the long hand that indicates the minutes.



The minute hand covers 360° when it completes its cycle, in other words, an hour. This angle is a **full angle**. It is a total of angles covered in 60 minutes of an hour, so after each minute, the minute hand covers 6° and their sum form a complete angle of 360 degrees.

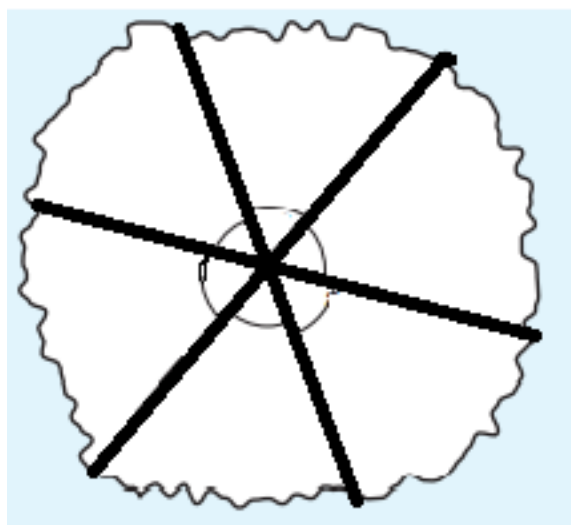
b) Objective:

Verify that the sum of angles formed around a point is 360° .

c) Required materials

- Manila paper, protractor, geoboard, ruler, markers, pencils, etc

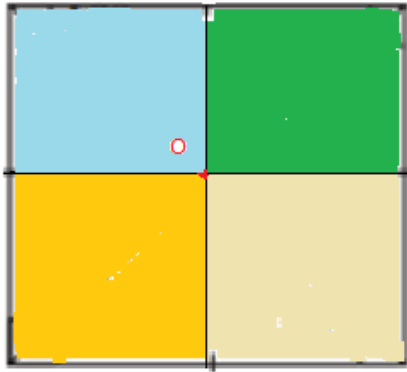
d) Activity set up (illustration)



e) Procedures

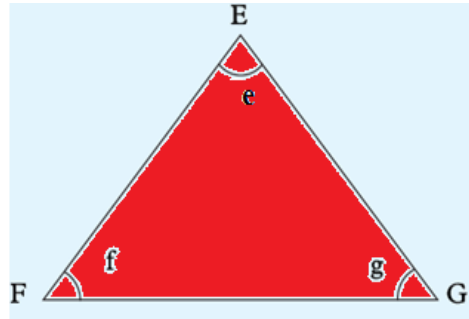
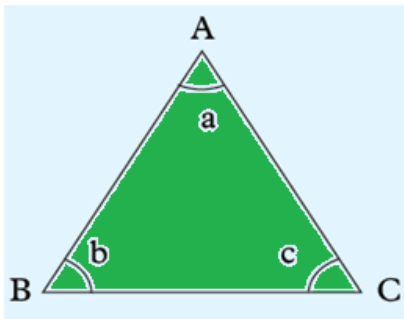
Step 1: Fold a paper into two equal parts,

Step 2: In the same way, fold it again. What do you observe?



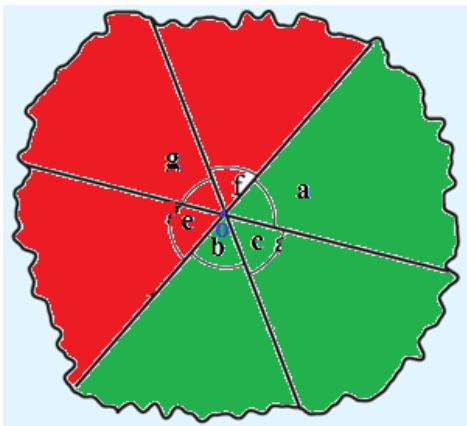
Step 3: Unfold the paper and measure the angles from their common vertex. What do you notice? what is the sum of 4 angles at the point O?

Step 4: Draw and cut out two triangles of any measurement as shown below:



Step 5: On a plain piece of paper, mark a point O

Step 6: Cut off all the angles of triangles ABC and EFG. Fit all the six angles on the point O, adjacent to each other without overlapping as shown in the figure below.



i) What do you notice? Discuss.

- ii) What is the sum of all the six angles a, b, c, e, f, and g?
- iii) What is the sum of angles at any point?

f) Results, their interpretation and Conclusion

From step3: The sum of 4 angles at point O is $90^\circ + 90^\circ + 90^\circ + 90^\circ = 360^\circ$

From step 6:

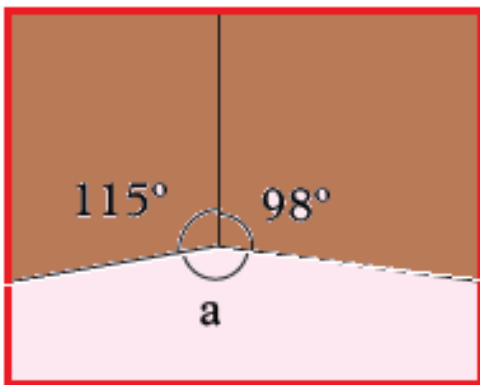
- i. When folding a paper twice and then after unfolding it, you find 6 angles with the common point or centre. The angles formed at the common point or at centre add up to 360° .
- ii. Angles a, b, c, d, e, f and g are centre angles at point O $a + b + c + d + e + f + g = 360^\circ$.

g) Information for teachers

- i. Angles that share a common vertex and add up to 360° are called **angles at a point or central angles**.
- ii. The angles at the common point add up to 360° .

h) Guidance on the evaluation

Ask students to help a carpenter to find the angle marked with letter **a** on a part of a timber.



Expected answer:

$$115^\circ + 98^\circ + a = 360^\circ \text{ (angles at Centre add up to } 360^\circ\text{)}$$

$$213^\circ + a = 360^\circ$$

$$a = 360^\circ - 213^\circ = 147^\circ$$

$$\therefore a = 147^\circ$$

PRACTICAL ACTIVITY/ EXPERIMENT 9: Forming different shapes on a geoboard and exploring their areas

a) Rationale:

This experiment is done when introducing the surface area of solids. It is taught in the unit 7 of S1.

In real life, the windows, doors, bed, chairs, TVs, mats, rugs, cushions, etc. all have different shapes. Moreover, beds sheets, quilts, covers, mats, and carpets have different geometric patterns on them. Therefore, it is important to learn geometric shapes because we are surrounded by them without being aware of them. .

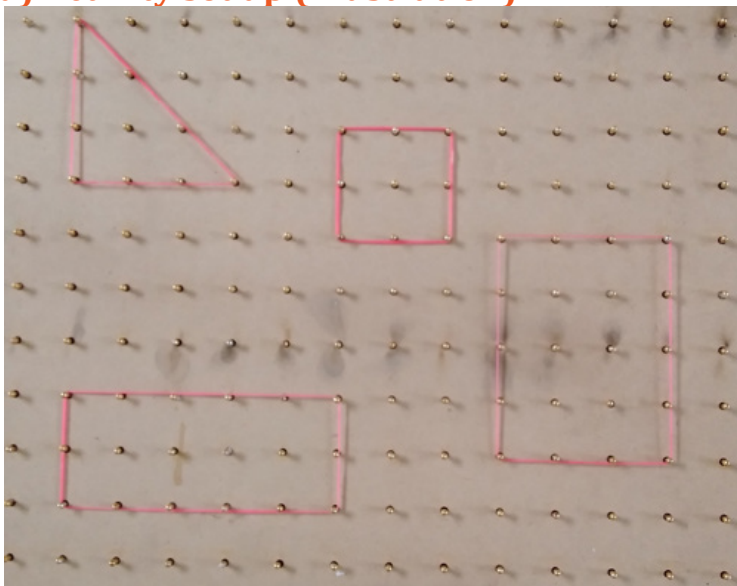
b) Objective:

To form different shapes on a geoboard and explore their areas.

c) Required materials

Geoboard
Rubber bands
Geoboard pins

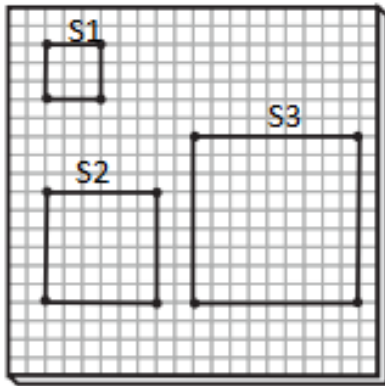
d) Activity set up (illustration)



e) Procedures

Step 1:

- i) Form the shapes of different squares using geoboard pins and rubber bands on a geoboard as shown in the figure.



ii) Count the number of unit squares enclosed in each of the three square and complete the following table.

| S. NO. | Total number of unit squares in the square | Side of the square | Side \times Side |
|--------|--|--------------------|--------------------|
| S1. | | | |
| S2. | | | |
| S3. | | | |

Compare the total number of unit squares and the product Side \times Side . What do you notice? What can you conclude about the area of a square?

Expected answer:

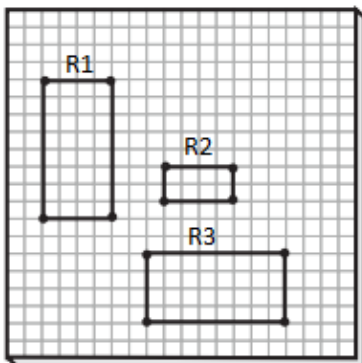
The number of unit squares is equal in each side of square.

| S. NO. | Total number of unit squares in the square | Side of the square | Side \times Side |
|--------|--|--------------------|--------------------|
| S1. | 9 | 3 | 9 |
| S2. | 36 | 6 | 36 |
| S3. | 81 | 9 | 81 |

we see that the total number of unit squares equals side \times side.

Step 2:

i) Form shapes of different rectangles using geoboard pins and rubber bands on the geoboard as shown in Figure:



ii) Now count the number of unit squares enclosed in each rectangle and complete the following table:

| S.NO. | Total number of unit squares in rectangle | Length of the rectangle | width of the rectangle | Length \times width |
|-------|---|-------------------------|------------------------|-----------------------|
| 1. | | | | |
| 2. | | | | |
| 3. | | | | |

Compare the total number of unit squares in the entire rectangle and the product of “Length \times Width”.

- i. What do you notice?
- ii. What can you conclude about the area of a rectangle?

Expected answer:

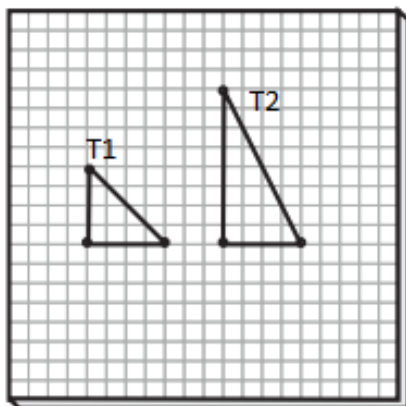
The number of unit squares of each rectangle is equal to the product “Length \times Width”.

| S.NO. | Total number of unit squares in rectangle | Length of the rectangle | width of the rectangle | Length \times Width |
|-------|---|-------------------------|------------------------|-----------------------|
| 1.R1. | 48 | 12 | 4 | 12 \times 4 |
| 2.R2. | 8 | 2 | 4 | 4 \times 2 |
| 3.R3. | 48 | 12 | 4 | 12 \times 4 |

We see that the total number of unit squares equals $L \times a$.

Step 3:

- i) Form shapes of different right-angled triangles with the help of geoboard pins and rubber bands on a geoboard as shown in the following figure:



- ii) Now count the number of unit squares enclosed in each right-angled triangle using the criterion for finding the area of any figure by counting the number of unit squares enclosed in it and complete the following table:

| S.NO | Total number of unit squares in the right triangle | Height (h) | Base (b) | $\frac{1}{2}(h \times b)$ |
|------|--|------------|----------|---------------------------|
| T1 | | | | |
| T2 | | | | |

Compare the total number of unit squares in the entire rectangle and the product " $\frac{1}{2}(\text{base} \times \text{height})$ " What do you notice? What can you conclude about the area of a triangle?

Expected answer:

| S. NO. | Total number of unit squares in the right triangle | Height (h) | Base (b) | $\frac{1}{2}(h \times b)$ |
|--------|--|------------|----------|---------------------------|
| 1.T1 | 8 | 4 | 4 | 8 |
| 2.T2 | 16 | 8 | 4 | 16 |

We see that the total number of unit squares equals $\frac{1}{2}(h \times b)$

f) Results interpretation and Conclusion

Basing on the findings, to determine the area of the triangles using a geoboard, we count the number of unit squares enclosed in it in the following way:

- i. Count one complete unit square enclosed by the figure and take its area as 1 square unit.
- ii. Count the unit square which is more than half enclosed by the figure and take its area as 1 square unit.
- iii. Count the unit square which is half enclosed by the figure as $1/2$ and take its area as $1/2$ square unit.
- iv. Neglect the unit squares which are less than half enclosed by the figure.

g) Additional information for teachers

- i. The area of a square is the product Side \times Side
- ii. The area of a rectangle is the product Length \times Width .
- iii. The area of a triangle is $\frac{1}{2}(\text{base} \times \text{height})$

h) Guidance on the evaluation

Invite students to form a right-angled triangle that has an area of 100cm^2 and a base of 10cm on a geoboard. Ask them to find the height of that triangle.

PRACTICAL ACTIVITY 10: Verifying Euler's rule by counting the numbers of faces and vertices for a solid

a) Rationale:

This activity can be done when teaching the properties of Solids. It is taught in unit 7 of S1.

Almost each and everything around us can be considered a 3D shape. These objects surround our daily lives. A lot of times, they go unnoticed. The laptop, phone, or tablet have solid shape. Similarly, the chair, bench, or couch you sit on is also a 3D shape. Look around and see: Ice cream cone and funnel cone have the form of cone, Roofs of houses have the form of pyramid, some boxes and dice have the form of a cube, carton boxes and bricks have the form of cuboids, balls and oranges have the form of sphere. Therefore, there are many examples of objects around us that show us that we need to learn properties of 3D shapes.

b) Objective

Verify the Euler's rule: $f + v = e + 2$

where f is the number of faces, v the number of vertices and e the number of edges for a solid.

c) Required materials:

cardboard model, cartons, basic geometric solids, pens, pencils, rulers.

d) Illustration:



e) Procedures

Count the number of faces, edges and vertices. Hence, complete the table

| Solids | Number of faces | Number of vertices | Number of edges | F + v | E + 2 | F+v=e+2? |
|------------------|-----------------|--------------------|-----------------|-------|-------|----------|
| Cube | | | | | | |
| Cuboid | | | | | | |
| Triangle prism | | | | | | |
| Triangle pyramid | | | | | | |

Step 1:

- i) Take a cardboard of cube, observe and identify the parts of the cardboard
- ii) Count the number of faces, edges and vertices.

Step 2:

- i) Take a cardboard of a cuboid, observe and identify the parts of the cardboard
- ii) Count the number of faces, edges and vertices.

Step 3:

- i) Take a cardboard of triangle prism, observe and identify the parts of the cardboard
- ii) Count the number of faces, edges and vertices.

Step 4:

- i) Take a cardboard of triangle pyramid, observe and identify the parts of the cardboard
- ii) Count the number of faces, edges and vertices.

f) Results, interpretation and Conclusion

| Solids | Number of faces | Number of vertices | Number of edges | F + v | E + 2 | F + v = e + 2? |
|------------------|-----------------|--------------------|-----------------|-------|-------|----------------|
| Cube | 6 | 8 | 12 | 14 | 14 | Yes |
| Cuboid | 6 | 8 | 12 | 14 | 14 | Yes |
| Triangle prism | 5 | 6 | 9 | 11 | 11 | Yes |
| Triangle pyramid | 4 | 4 | 6 | 8 | 8 | Yes |

g) Information for teachers

- i. The intersecting line of two faces (f) of a polyhedron is called **an edge (e)**.
- ii. Edges meet at a point which is referred to as **a vertex (vertices) (v)**
- iii. The number of faces (f), edges (e) and vertices (v) of a solid are related by: **$f + v = e + 2$**

The sum of the number of faces and the number of vertices is equal to the number of edges plus 2

h) Guidance on the evaluation

Invite students to explore other polyhedrons for example rectangular prism, rectangular pyramid, etc.

PRACTICAL ACTIVITY 11: Exploring the concept of surface area of a cuboid by using folded up nets

a) Rationale:

This activity is done when teaching the surface area of a solid. It is taught in unit 7 of S1.

In real life, cuboid shapes are often used for boxes, cupboards, rooms, buildings, containers, cabinets, books, a sturdy computer chassis, printing devices, electronic calling touchscreen devices, washing and drying machines, etc. Cuboids are among those solids that can tessellate 3-dimensional space.

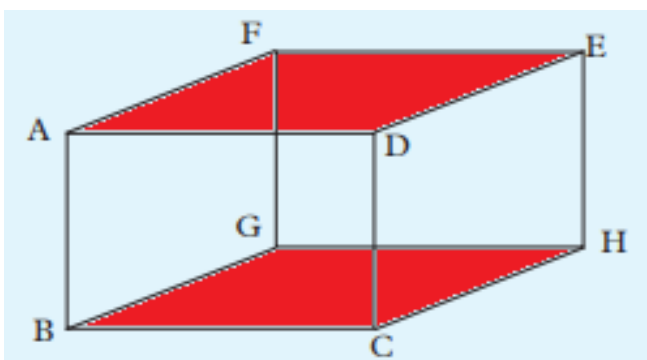
b) Objective:

To verify practically the use of folded up nets to calculate the surface area of a cuboid.

c) Required materials:

Cardboard model of cuboid, Cutter (scissors), Ruler, Pen, sheets of Paper.

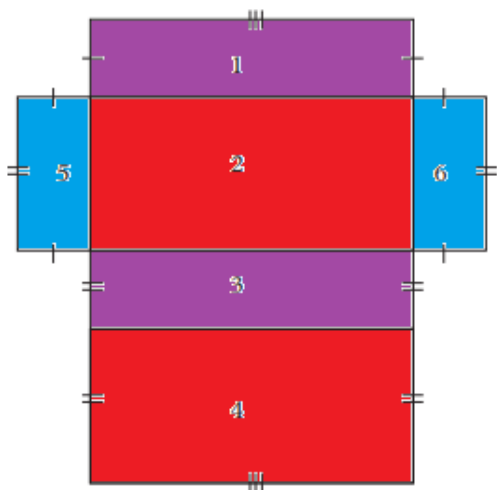
d) Illustration or diagram



e) Procedures

Step 1: Take a cardboard model of cuboid and observe it. How many faces does the cuboid have?

Step 2: Unfold a cuboid. How would you calculate the area of cuboid using the net?



Step 3: Use the net to calculate its area.

f) Interpretation of results and Conclusion

- i. The cuboid has 6 faces

The surface area of an opened cuboid or cuboid net is given by $A_1 + A_2 + A_3 + A_4 + A_5 + A_6$

Where A_1 = Area of rectangle 1

Where A_2 = Area of rectangle 2

Where A_3 = Area of rectangle 3

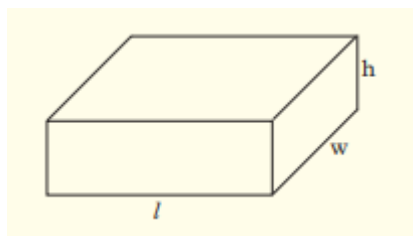
Where A_4 = Area of rectangle 4

Where A_5 = Area of rectangle 5

Where A_6 = Area of rectangle 6

g) Information for teachers

- i. The surface area of a cuboid of length l , width w , and height h , is given by $2lw + 2lh + 2wh$

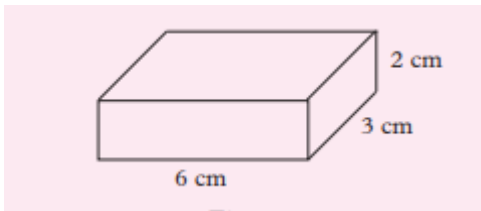


- ii. The sum of the area of the six faces gives the surface area of a cuboid.

h) Guidance on the evaluation

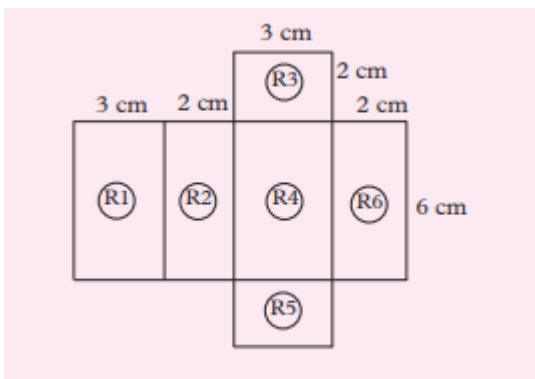
Ask students to find answers for some questions related to the determination of surface area of cuboids. For example, The net of a cuboid consists of a series of rectangles.

- i) How many rectangles are there?
- ii) What is the surface area of the cuboid if it measures 6 cm by 3 cm by 2 cm?



Expected answer:

This figure shows a possible net of a cuboid which measures 6 cm by 3 cm by 2 cm. It is composed of three pairs of rectangles i.e. 6 rectangles.



The surface area of the cuboid = sum of the areas of all the rectangles that comprise the net.

Area of rectangle R1 = $6 \text{ cm} \times 3 \text{ cm} = 18 \text{ cm}^2$

Area of rectangle R2 = $6 \text{ cm} \times 2 \text{ cm} = 12 \text{ cm}^2$

Area of rectangle R3 = $3 \text{ cm} \times 2 \text{ cm} = 6 \text{ cm}^2$

Area of rectangle R4 = $6 \text{ cm} \times 3 \text{ cm} = 18 \text{ cm}^2$

Area of rectangle R5 = $3 \text{ cm} \times 2 \text{ cm} = 6 \text{ cm}^2$

Area of rectangle R6 = $6 \text{ cm} \times 2 \text{ cm} = 12 \text{ cm}^2$

Total surface area = $18\text{cm}^2 + 12\text{cm}^2 + 6\text{cm}^2 + 18\text{cm}^2 + 6\text{cm}^2 + 12\text{cm}^2 = 72 \text{ cm}^2$

Surface area of the cuboid = 72 cm^2 .

PRACTICAL ACTIVITY 12: Exploring the concept of surface area of a prism by using folded up nets

a) Rationale:

This activity is done when teaching the surface area of a prism. It is taught in unit 7 in S1.

In real life, Prisms are defined as solid objects with flat sides, identical ends, and the same cross section throughout the entire length of the object. Prism-shaped objects we see in everyday life include ice cubes, outhouses, and candy bars. The regular geometry of the prism makes it useful for designing buildings and simple products. We also find prisms in mineral crystals.

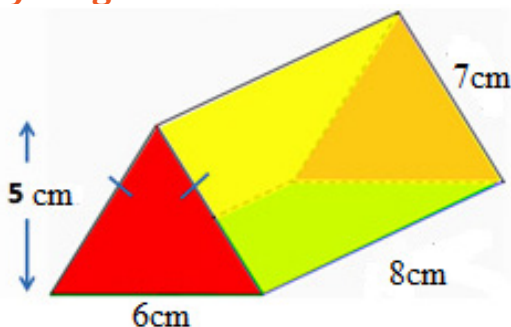
b) Objective:

To explore practically the surface area of a **triangular prism**.

c) Required materials

Cardboard model of prism, a pair of scissors, basic geometric solids, pens, pencils etc

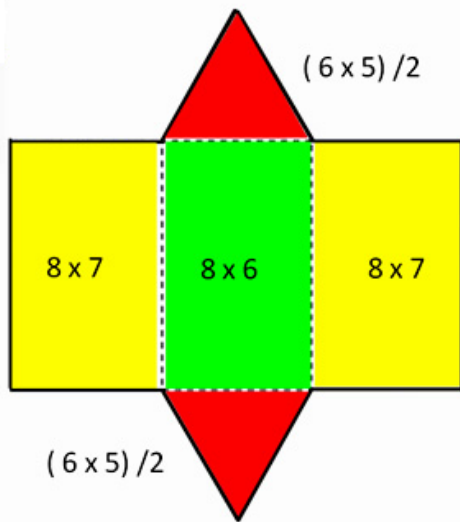
d) Diagram



e) Procedures

Step 1: Take a cardboard model of a prism and observe it. How many faces does the prism have?

Step 2: Unfold a prism. How many shapes can the cross-section take?



Step 3: Use the faces observed on the folded-up nets to calculate the area of each face.

Step 4: Discuss how you can get the area of all faces.

f) Interpretation of results and Conclusion

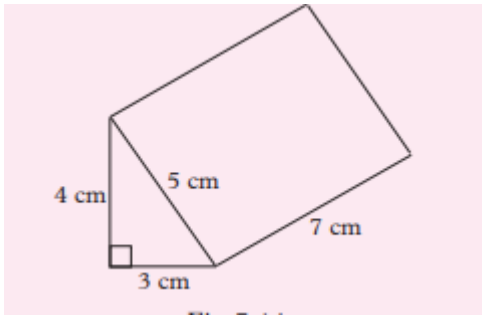
- i. The cross-section may take the shape of any polygon such as triangle, square, rectangle, hexagon etc.
- ii. The faces of a prism other than the cross-section (or the bases) are called **lateral faces** of the prism.
- iii. The edges joining pairs of lateral faces are called **lateral edges**.
- iv. A prism has two parallel bases.
- v. A prism has as many lateral faces as the number of sides of its base.
- vi. The bases of a prism are identical.
- vii. The lateral edges of a prism are equal and parallel.
- viii. The lateral faces of a prism are perpendicular to the bases.

g) Additional information for teachers

If the base of a prism has n sides, then the prism has n lateral faces and since any prism has two bases, the total number of faces is $2 + n$ lateral faces. Total number of faces = $(n + 2)$ faces. The total surface area of a prism = 2 (area of one base) + areas of the n lateral **rectangular** faces

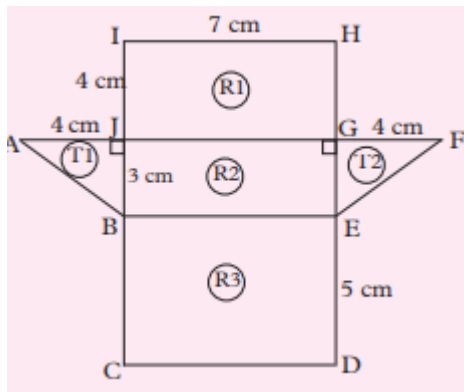
h) Guidance on the evaluation

Ask students to construct the net of the prism in Figure below and use it to find the surface area of the prism.



Expected answer:

Represent the net of the prism



The area of this prism is made up of 3 rectangles and 2 triangles

$$\text{Area of rectangle R1} = 7 \text{ cm} \times 4 \text{ cm} = 28 \text{ cm}^2$$

$$\text{Area of rectangle R2} = 7 \text{ cm} \times 3 \text{ cm} = 21 \text{ cm}^2$$

$$\text{Area of rectangle R3} = 7 \text{ cm} \times 5 \text{ cm} = 35 \text{ cm}^2$$

$$\text{Area of triangle T1} = \frac{1}{2} (3 \text{ cm} \times 4 \text{ cm}) = 6 \text{ cm}^2$$

$$\text{Area of triangle T2} = \frac{1}{2} (3 \text{ cm} \times 4 \text{ cm}) = 6 \text{ cm}^2$$

$$\text{Total surface area of the prism: } 28\text{cm}^2 + 21\text{cm}^2 + 35\text{cm}^2 + 6\text{cm}^2 + 6\text{cm}^2 = 96 \text{ cm}^2$$

PRACTICAL ACTIVITY 13: Exploring the surface area of a pyramid by using folded up nets

a) Rationale:

This activity is conducted when teaching the surface area of a Pyramid. It is taught in unit 7 of S1.

In real life, some items that feature a pyramid shape can be found inside the house and can be a part of a house itself. For example, the roof on a home often features a pyramid-type design. Camping tents, when erected, can take the shape of a pyramid. The package of a Toblerone chocolate bar has a pyramid design. A cheese grater and a waffle cone resemble a pyramid.

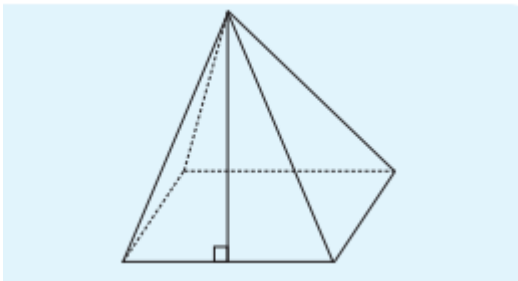
b) Objective:

To use folded up nets to calculate the surface area of a pyramid with a **squared base**.

c) Required materials

Cardboard model of pyramid or the solid of pyramid and related folded up nets
Cutter (pair of scissors), Manila paper, pens, pencils or markers

d) Illustration or diagram



e) Procedures

Step 1. Take a cardboard model of a pyramid and observe.

Step 2. Unfold the pyramid and observe it: How many identical faces does the pyramid have?

Step 3. Use the nets to calculate the total area of faces.

- i) How do we get the area of one slant surface? How many slant faces are there? Are all slant faces equal?
- ii) How do we get the area of the base?
- iii) How do we get the total area of the pyramid?

f) Results interpretation, and Conclusion

- i. The altitude of a slant face of a pyramid is called a slant height of the pyramid. It is perpendicular to an edge of the base.

ii. Surface area of a pyramid = total area of the slant faces + the area of base.

g) Information for teachers

- i. If the vertex of a pyramid is vertically above the centre of the base, the pyramid is called a right pyramid.
- ii. The net of a pyramid is composed of 4 identical slant faces.
- iii. Total surface area = 4(area of one slant face) + area of the base.

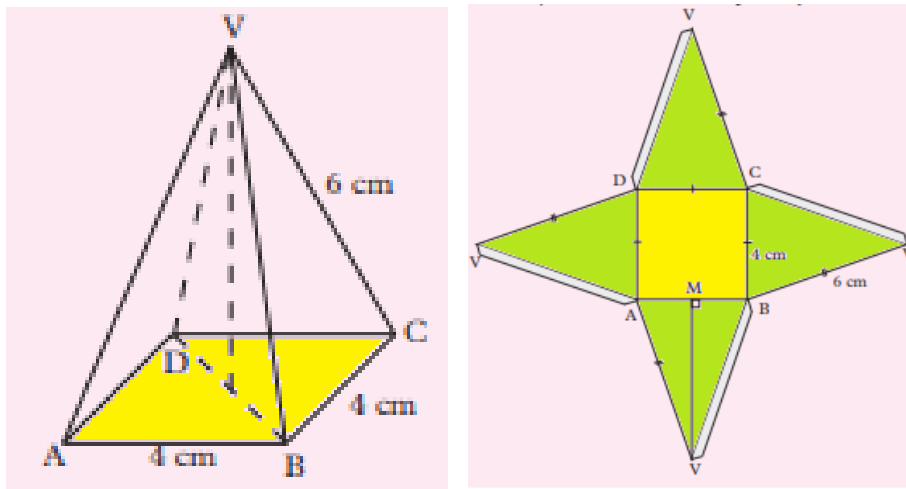
h) Guidance on the evaluation

The base of a right pyramid is a square of sides 4 cm. The slant edges are all 6 cm long.

- a) Draw and label and **make** a sketch of the solid using the **Manila paper and scotch**.
- b) Draw a net of the pyramid.
- c) Cut and use the net to find:
 - i) The slant height of the pyramid.
 - ii) The total surface area of the pyramid.

Expected answer:

Let the base of the pyramid be ABCD, and its vertex be V. its net.



(c) (i) The inner segment VM on the net in part (b) represents the slant height of the pyramid.

By measurement $VM = 5.7$ cm.

(ii) Total surface area of the pyramid = $(\frac{1}{2} \times 4 \times 5.7) \times 4 + (4 \times 4) = 45.6 + 16 = 61.6$ cm².

PRACTICAL ACTIVITY 14: Exploring the concept of surface area of a cylinder by using folded up nets

a) Rationale:

This activity is done when teaching the surface area of a cylinder. It is taught in unit 7 of S1.

In real life, a cylinder is a three-dimensional object with two circular bases that run parallel to one another. The circular bases are connected by one curved side made up of parallel lines that run between the points along the edges of the circular bases. There are many examples of cylinders in the world around us and we use them throughout our daily lives: Pipes, Beaker, Cold drink cans, Water tanks, Gas cylinder, Candle, Fire extinguisher, etc.

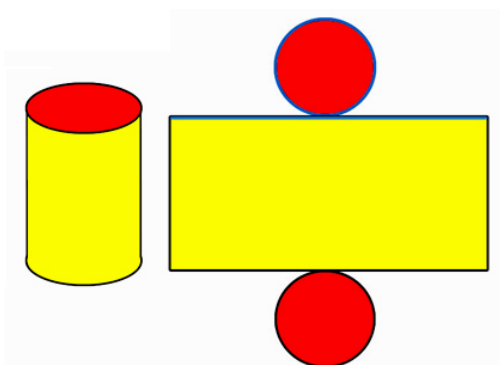
b) Objective:

To use of folded up nets and practically verify the surface area of a cylinder.

c) Required materials

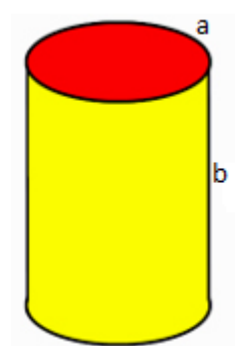
Manila paper, paper glue, ruler, scissors, markers, etc.

d) Illustration or diagram

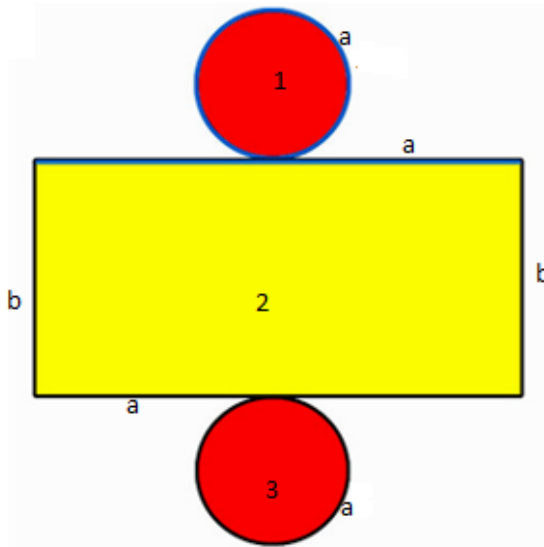


e) Procedures

Step 1: Take a cylindrical object of height " b " and covered by two circular bases of circumference " a ".



Step 2: Now unfold or open the cylinder along its height, and make sure that the bases are opened separately by lateral part.



Step 3. Use the obtained net to calculate its area.

f) Interpretation of results and Conclusion

From the net, it shows that lateral face of cylinder becomes rectangle and its bases become circles.

- i. Circular base becomes the length of the rectangle. i. e circumference of the base of cylinder = length of rectangular part = a
- ii. Since, the circumference of the base of cylinder is given by $2r\pi$, then $2r\pi = a = l$
- iii. The height (h) of the cylinder = breadth of rectangular paper = b
- iv. The area of rectangle is $A_2 = a \times b = 2r\pi \times b = 2r\pi b$
- v. The area of circular first base is $A_1 = r^2\pi$
- vi. The area of another base $A_3 = r^2\pi$
- vii. Total area of the net $A_T = A_1 + A_2 + A_3 = r^2\pi + 2r\pi b + r^2\pi$

$$= 2r^2\pi + 2r\pi b$$

$$= 2\pi r(r + b)$$

g) Addition information for teachers

- i. As folding the rectangle, it becomes cylinder such that the curved surface area of cylinder is equal to the area of rectangle.
- ii. Now curved area of cylinder = area of rectangle $l \times w = (2r\pi) \times h = 2r\pi h$
- iii. The area of base is $r^2\pi$
- iv. The surface area of a closed cylinder is composed of the sum of the curved surface and the two end faces.

NOTE: The total surface area is:

$$\text{Open one end cylinder} = \pi r^2 + 2\pi rh$$

$$\begin{aligned}\text{Closed cylinder} &= 2\pi r^2 + 2\pi rh \\ &= 2\pi r(r + h)\end{aligned}$$

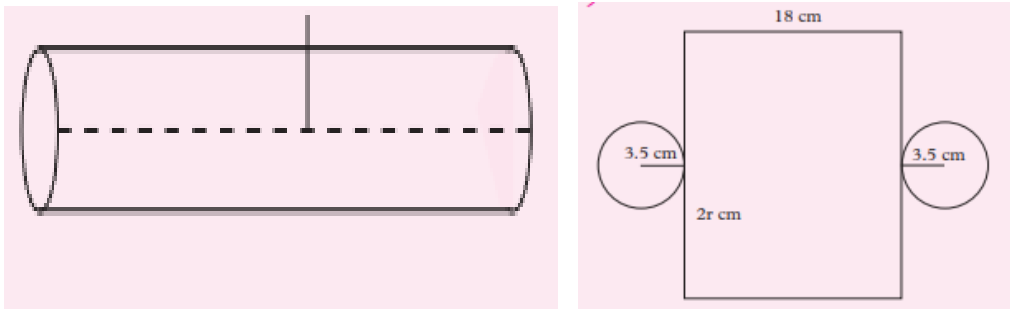
h) Guidance on the evaluation

Ask students to use their findings to solve the following problem:

A closed cylinder whose height is 18 cm has a radius of 3.5 cm. Draw the net of the cylinder and use it to find the total surface area of the cylinder.

Expected answer:

Figure of the diagram.



The circumference of one of the end face (circle) = $2\pi r = 22$ cm

The net consists of two end circles, and a rectangle measuring 22 cm by 18 cm.

Total surface area

$$\begin{aligned}&= 2 \times \pi r^2 + (2\pi r) h \\ &= 2 \times \pi \times (3.5 \text{ cm})^2 + 22 \text{ cm} \times 18 \text{ cm} \\ &= (76.98 + 396) \text{ cm}^2 \\ &= 472.98 \text{ cm}^2\end{aligned}$$

PRACTICAL ACTIVITY/ EXPERIMENT 15: Formula of the volume of a cylinder

a) Rationale:

This experiment is done when exploring the volume of a cylinder. It is taught in unit 7 of S1.

There are many examples of cylinders in the world around us and we use them throughout our daily lives: Pipes, Beaker, Cold drink cans, Water tanks, Gas cylinder, Candle, Fire extinguisher, etc. By observing a cylindrical object, it is not easy to guess how its volume is determined. It is necessary to learn a formal way of calculating the such a volume.

b) Objective:

To show how you can derive the volume of a cylinder using real material.

c) Required materials:

Wooden Cylinder cut into 12 pieces.

d) Illustration or diagram



e) Procedures & Steps of experiment:

Step1: Take and join pieces of wood such that the same-colored pieces do not be placed together. (make sure that different colored pieces are alternatively joined)



- i. What solid do you obtain?
- ii. What is the shape of the base for this solid?
- iii. Approximate the formula for the Volume of the solid.

Step 2: Repeat the process of placing 12 pieces together in alternative manner for example as illustrated below.



- i. What solid do you obtain?
- ii. What is the shape of the base does this solid has?
- iii. Approximate the formula for the Volume of the solid.

f) Interpretation of results

From this step 1, we have

- i. The wooden cylinder with the circular base as shown in the figure above.
- ii. The segments approximately form a solid cylinder of height 'h', and radius 'r'.

From this step 2, we have

- iii. The wooden cuboid with rectangular base as shown in the figure above.
- iv. The segments approximately form a solid cuboid of height 'h', and base with breadth 'r' and length ' πr '.

Its volume is the volume of cuboid $V=L \times b \times h = \pi.r.r.h$

$$V = \pi.r^2.h$$

g) Additional information for teachers

- i. The volume of a solid is a measure of the amount of space occupied by that solid.
- ii. The volume of the cylinder is the area of the base ($r^2\pi$) times the height (h).
 $V = \pi r^2 h$; where r is the radius of the base and h is the height.

- iii. It should be possible that student's alternative ideas contain errors/ misconception related to the experiment.
- iv. The teacher should highlight that the height of cylinder is the straight line joining the two bases of cylinder and perpendicular to them.

h) Guidance on the evaluation

Ask students to use their findings to work out the following problem.

A cylindrical mug has inner diameter 9.2 cm and an inner height of 12.5 cm. Taking π as 3.142, find its volume

Expected answer:

Volume of cylinder is $= \pi r^2 h$.

Capacity of the mug $= (3.142 \times 4.6 \times 4.6 \times 12.5) \text{ cm}^3 = 831.1 \text{ cm}^3$

PRACTICAL ACTIVITY 16: Data organisation/ collection and frequency distribution table

a) Rationale:

This activity is done when teaching the organization of data in statistics using frequency distribution table. It is taught in unit 8 of S1.

In real life of business, good data organization strategies are important because your data contains the keys to managing your company's most valuable assets. Getting insights out of this data could help you obtain better business intelligence and play a major role in your company's success.

b) Objective:

To be able to collect, represent, and interpret data using frequency distribution table.

c) Required materials

Marked papers for exams or test, manila paper, markers, pencils.

d) Procedures

Step 1: Distribute the marks scored by 40 students of P6.

Step 2: Observe their marks dispatched:

78 46 55 47 77 63 52 52 62
46 77 47 40 35 67 61 58 52
42 40 48 57 66 54 75 78 75
59 75 47 59 35 62 53 72 57
51 69 55 57

Step 3: Organize papers from the one with few marks by putting together the papers with the same marks.

| Order | Mark | Number of copies with the same mark |
|-------|------|-------------------------------------|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| ... | | |

How such data can be collected?

How do we call the number of copies with the same mark?

Is there another name to give to the highest mark?

Step 4: Discuss how you can answer to the following questions:

- What is the highest score?
- What is the lowest score?
- What is the difference between the highest score and the lowest score?
- What mark was scored by most students?
- How many students scored above 70?
- If the pass mark was 45, how many students failed the test?
- How many students scored between 30 and 39
- What about your findings?

Expected answers:

- The highest score is 78
- The lowest score is 35
- The difference between highest and lowest data is $78-35=43$
- The marks scored by most students are 40, 47, 52, 57, 75.
- If the pas mark is 45; 6 students are failed
- The student scored between 30 and 35 are 2

e) Interpretation of results and Conclusion

- All the findings in both questions are called **data collection**.
- The number of times the score occurs is called **frequency**.
- The score that has the highest frequency is **mode**
- The difference between highest and lowest mark is called **range**.

f) Additional information for teacher

In a given data distribution,

Data collection: is the process of gathering/collecting, measuring and analyzing information for research using standard validated method.

Frequency: is the number of times an item or value occurs.

Mode: is the value or the item that has the highest frequency/repetition.

Range: is the difference between highest and lowest value.

g) Guidance on the evaluation

The following data express the length in cm of 20 pupils who study in P3 at a certain school.

71, 71, 72, 75, 73, 75, 76, 76, 75, 72, 78, 79, 75, 78, 79, 75, 71, 73, 75, 76.

Organize them in a **frequency table**. What is the mode of the data?

Expected answer:

| Numbers | Frequency |
|----------------|------------------|
| 71 | 3 |
| 72 | 2 |
| 73 | 2 |
| 75 | 6 |
| 76 | 3 |
| 78 | 2 |
| 79 | 2 |
| TOTAL | 20 |

The length of 75 has the highest frequency. A big number of pupils has the length of 75cm. This means that 75 is the mode of these data.

PRACTICAL ACTIVITY 17: Constructing Bar chart of a discrete data using real material

a) Rationale:

This activity is done when teaching the presentation of data in statistics using Bar chart of a discrete data. It is taught in unit 8.

A bar diagram makes it easy to compare sets of data between different groups at a glance. The graph represents categories on one axis and a discrete value in the other. The goal is to show the relationship between the two axes. Bar charts can also show big changes in data over time.

b) Objectives:

To show how to construct the bar chart of a discrete data using real material.

c) Materials:

Manila paper, marker, meter rule, sticks, scissors, adhesive, colored paper with different colors.

d) Procedures & Steps of experiment:

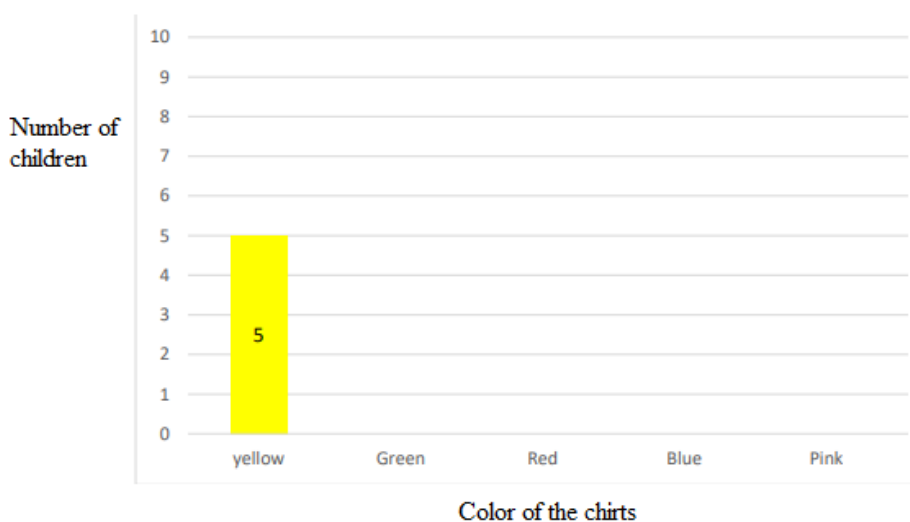
Step1: Draw accurate Cartesian plane on a manila paper.

Step2: Mark the data on the x -axis and frequency on the y -axis with a marker.

Step3: Cut a colored paper into rectangular piece or take a stick with height equal to the frequency of each data given in the situation below:

“Consider a class of 27 pupils who have shirts with different colors. 5 children with yellow shirts, 7 with green, 4 with red, 9 with blue, and 2 with pink shirts”.

Step 4: In the graph, Glue with adhesive each piece of paper or a stick in appropriate position/place following the example of yellow shirts.

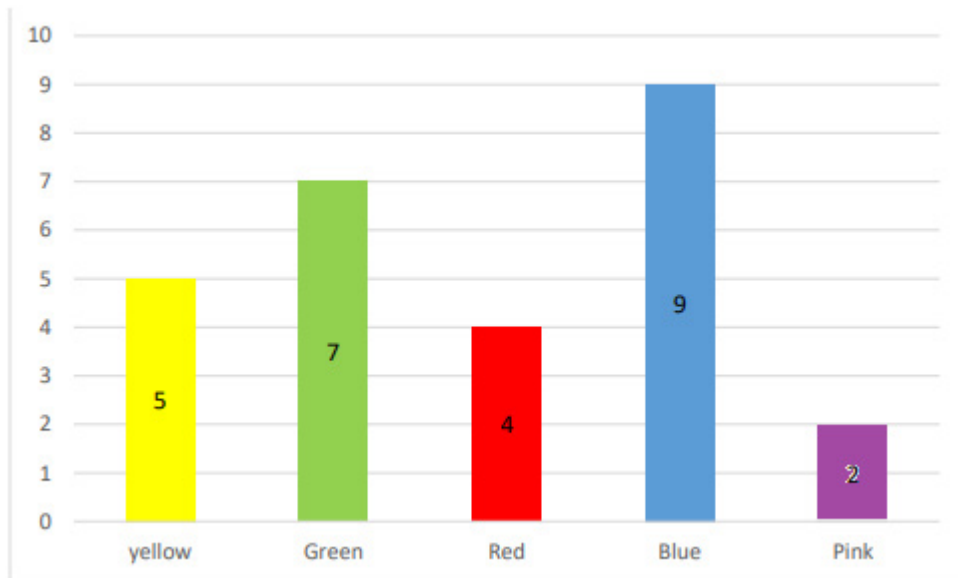


How do you call this type of data presentation?

e) Interpretation of results

By gluing each piece of paper to the corresponding data, we obtained a bar chart.

Each bar has the height equal to the frequency of the corresponding data.



f) Additional information to the teachers

A bar chart (or bar graph) is a graph consisting of rectangular bars whose lengths are proportional to the frequencies in a data distribution.

g) Guidance on the evaluation

Ask students to do use their findings to work out the following problems:

- 1) There are sizes of sweaters worn by 30 Senior 1 students in a certain school.

| size | Small (S) | Medium(M) | Large(L) | Extra-large(XL) |
|-------------------------|-----------|-----------|----------|-----------------|
| Number of pupils | 5 | 13 | 8 | 4 |

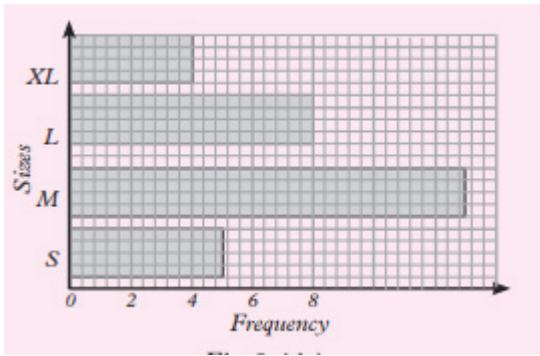
Represent the data on:

- a) a horizontal bar chart
- b) a vertical bar chart.

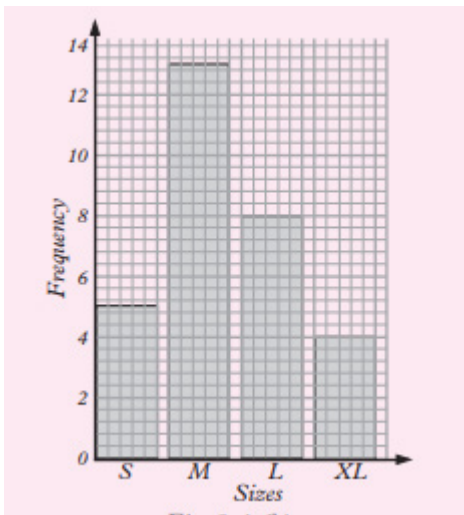
Expected answer

(a) Horizontal bar chart

- (a). In a horizontal bar chart, frequency is represented on the horizontal axis. Bars are drawn with spaces between them, and they may be shaded or not.



(b) Vertical bar chart in a vertical bar graph, frequency is represented on the vertical axis.



PRACTICAL ACTIVITY 18: Constructing histogram and frequency polygon of a discrete data using real material

a) Rationale:

This activity is done when teaching the presentation of data in statistics using **histogram and frequency polygon of a discrete data**. It is taught in unit 8 of S1.

The histograms are mainly used to display and organize a large set of measurements or numerical data in a user-friendly manner. A histogram will make it easy to see where the majority of values fall on a measurement scale, and how much variation is there among those values.

b) Objectives:

To show how to construct the histogram and **frequency polygon** of a discrete data using real materials.

c) Materials:

Manila paper, marker, meter ruler, sticks, scissors, adhesive, colored paper with different colors.

d) Procedures & Steps of experiment:

Step 1: Draw accurate Cartesian plane on a manila paper.

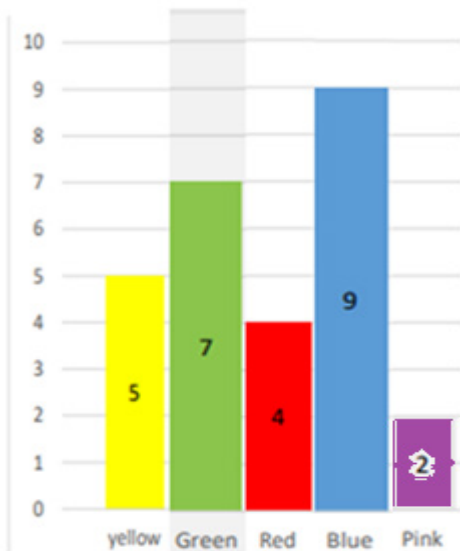
Step 2: Mark the data on the x -axis and frequency on the y -axis with a marker.

Step 3: Using a suitable scale, say vertical scale 1cm: 2 units and horizontal scale 1cm:2unit,

Step4: Cut a colored paper into rectangular piece with height equal to the frequency of each data below and breath equal to 1cm.

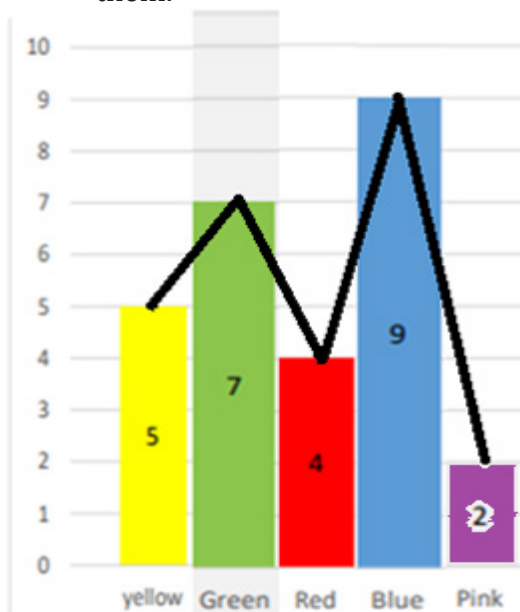
“Consider a class of 27 pupils who have shirts with different colors. 5 children with yellow shirts, 7 with green, 4 with red, 9 with blue, and 2 with pink shirts”.

Step 4: In the graph, Glue with adhesive each piece of paper in appropriate position/place by avoiding a space between pieces of papers.



How do you call this type of data presentation?

Step 5: From the histogram graph, identify and mark centres of the rectangular papers and use markers to highlight them. Then after, use sticks to join them.



How do you call this type of data presentation?

e) Interpretation of results

By gluing each piece of paper to the corresponding data, we obtained a histogram as shown in step 4 and polygon as shown in step 5.

f) Additional information for teachers

- A **histogram** is a diagram used to represent frequency distribution in an ungrouped data. A histogram resembles a bar graph or bar chart with the bars touching one another.
- A **frequency polygon** is a line graph drawn by joining all the midpoints of the top of the bars of a histogram.
- A teacher should insist on the difference between a Bar chart and a Histogram.

A bar graph is the graphical representation of categorical data using rectangular bars where the length of each bar is proportional to the value they represent while a **histogram** is the graphical representation of data where data is grouped into continuous number ranges and each range corresponds to a vertical bar (A histogram resembles a bar graph or bar chart with the bars touching one another).

g) Guidance on the evaluation

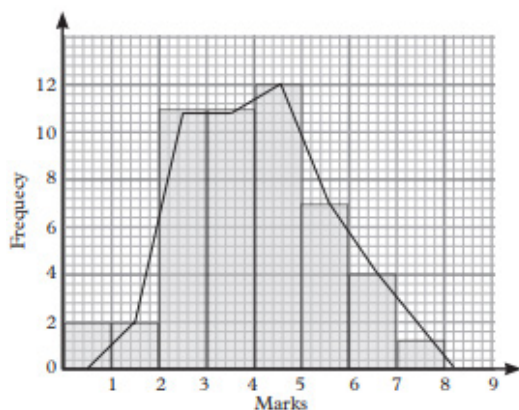
This table shows how a group of 50 students preformed in a maths quiz marked out of ten

| | | | | | | | | | | |
|-----------|---|---|----|----|----|---|---|---|---|----|
| Mark | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| frequency | 2 | 2 | 11 | 11 | 12 | 7 | 4 | 1 | 0 | 0 |

- Using a suitable scale, say vertical scale 2cm: 4 units and horizontal scale. 1cm:1unit, represent this information on a histogram.
- On the same graph, draw a frequency polygon.

Expected answer:

Since this set represents discrete elements which are ungrouped, all the rectangles will be of the same width, and the frequencies will be represented by the heights of the rectangles.



PRACTICAL ACTIVITY 19: Determining the probability of an event happening when tossing coins

a) Rationale:

This activity is done when teaching the probabilities for equally likely events. It is taught in unit 9 of S1.

Probability plays a vital role in the day-to-day life. In the weather forecast, sports and gaming strategies, buying, or selling insurance, online shopping, and online games, determining blood groups, and analysing political strategies.

b) Objective:

Determine the probability of an event happening when tossing a coin twice.

c) Required materials

Rwandan coins

d) Illustration or diagram



e) Procedures

Step 1. Obtain a Rwanda coin and let the side with the crown be the head (H) and the other side the tail (T).

Step 2: Throw the coin to the ground at the first time and let your partner record the side that comes up as the result, a head or a tail.

Step 3: Throw the coin to the ground at the second time and let your partner record the side that comes up as the result, a head or a tail.

Complete the table to record the outcomes of this experiment.

| | First throw | Second throw | Outcomes |
|---|-------------|--------------|----------|
| | H | T | |
| H | | | |
| T | | | |

- How many ways of getting 2 heads did you obtain?
- How many ways of getting 2 tails?
- How many ways of getting a head and a tail?
- What is the probability of getting:
 - i) 2 heads?
 - ii) 2 tails?
 - iii) 1 head and 1 tail?

f) Interpretation of results

- iv. If the coin shows head at the first time and head at the second time, we indicate the outcome as HH.
- v. If the coin shows H at once and the second time shows tail, we indicate the outcome as HT, etc.
- vi. So, all the possible outcomes are HH, HT, TH, TT.
- vii. Number of all possible outcomes = 4
- viii. Number of outcomes with two heads = 1
- ix. Number of outcomes with 1 head and 1 tail = 2
- x. Number of outcomes with two tails = 1

Hence,

$$(a) P(2 \text{ heads}) = P(HH) = 1/4 .$$

$$(b) P(1 \text{ head, 1 tail}) = P(HT) = 2/4 = 1/2 .$$

$$(c) P(2 \text{ tails}) = P(TT) = 1/4 .$$

g) Additional information for teachers

- i) An experiment** is any activity or process through which data is obtained and analysed.
- ii) Possible outcomes** are all likely results of an experiment.
- iii) A sample space** is a set of all possible outcomes that may occur in a particular experiment, usually denoted by S.
- iv) An event** is a set consisting of possible outcomes of an experiment with the desired qualities.

h) Guidance on the evaluation

Consider an event of tossing a coin three times

- i) Write down the sample space.
- ii) What is the probability of getting:
 - 1) 3heads?
 - 2) 2heads and 1tail?
 - 3) 2tails and 1head?

Expected result:

- i) When a coin is tossed thrice: $S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$

Note that H= Head and T=Tail then S=Sample space/outcomes.

$$(a) P(3 \text{ heads}) = P(HHH) = 1/8.$$

$$(b) P(2 \text{ head, 1 tail}) = P(HH,T) = 3/8$$

$$(c) P(2 \text{ tails, 1head}) = P(TT,H) = 3/8.$$

PRACTICAL ACTIVITY 20: Determining the probability of an event happening when playing cards

a) Rationale:

This activity is done when teaching the probabilities for equally likely events and deducing the chances of getting a particular outcome. It is taught in unit 9 of S1.

In real life, **Card games can teach math and memory skills, as well as strategic thinking**, psychologist and sociologists say. Also, the conversation and friendly rivalry that come with sitting down to play cards can strengthen family ties.

b) Objective:

Determine the probability of an event happening when playing cards.

c) Required materials

A pack of playing cards.

d) Illustration or diagram



e) Procedures

Step 1: Obtain a pack of 52 playing cards.

Step 2: Pick a card at random and let your partner record the card that comes up as the result, a heart, a spade, a club, a diamond, a red card, a black card, or an ass, etc.



- 1) How many ways to obtain a red card?
- 2) How many ways of getting a spade?
- 3) How many ways of obtaining an ass?
- 4) What is the probability of drawing:
 - i) A heart?
 - ii) A diamond?
 - iii) A spade?
 - iv) A club?
 - v) An ass?

f) Interpretation of results

From question 1)

In a pack or deck of 52 playing cards, they are divided into 4 suits of 13 cards each i.e. spades ♠ hearts ♥, diamonds ♦, clubs ♣.

Cards of Spades and clubs are black cards.

Cards of hearts and diamonds are red cards

Therefore, there are 26 reds and 26 blacks.

So when a card is picked, it can be a red or a black card, that is why there are 26 ways of obtaining a red card.



From question 2)

Since in a pack of playing cards, there are 52 cards of which 13 are spades, 13 clubs, 13 diamonds and 13 hearts. So when a card is picked, it can be a spade, a club, a diamond or a heart. That is why 13 ways are possible to get a spade.

From question 3)

Since in a pack of playing cards, we obtain only 4 asses, the we have only 4 ways to pick an ass.

From question 4) probability of drawing:

i) $P(\text{a heart}) = \frac{13}{52} = \frac{1}{4}$

ii) $P(\text{a diamond}) = \frac{13}{52} = \frac{1}{4}$

iii) $P(\text{a spade}) = \frac{13}{52} = \frac{1}{4}$

iv) $P(\text{a club}) = \frac{13}{52} = \frac{1}{4}$

v) $P(\text{a club}) = \frac{4}{52} = \frac{1}{13}$

g) Additional information for teacher

Teacher should have to set clear instructions.

Teacher should manage students' attitudes when playing cards for instance, advising them against gambling, let them use appropriate words etc

h) Guidance on the evaluation

In an experiment of drawing a card from a deck of 52 cards, what is the probability of drawing a black card?

Expected result

There are 26 black cards in a park. So the probability of getting a black card is $P(B)$.

This can be obtained as $P(B) = \frac{26}{52} = \frac{1}{2}$

PRACTICAL ACTIVITY 21: Determining the probability of events from throwing a die

a) Rationale:

This activity is done when teaching the probabilities for equally likely events. It is taught in unit 9 of S1.

Dice (singular **die**) are small, throwable objects with marked sides that can rest in multiple positions. They are used for generating random numbers, commonly as part of tabletop games, including dice games, board games, role-playing games, and games of chance.

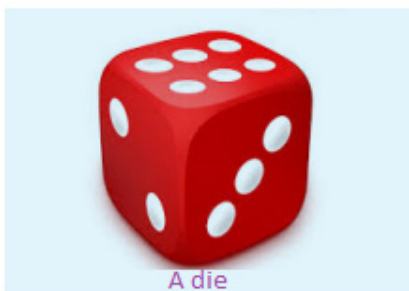
b) Objective:

Determine the probability of an equally likely event resulting from throwing a die.

c) Required materials

A dice, and template sheet.

d) Illustration or diagram



| At once \ At twice | 1 | 2 | 3 | 4 | 5 | 6 |
|--------------------|---|---|---|---|---|---|
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |

Template sheet

e) Procedures

Step 1: Obtain a die, observe it and record the number represented by the dots on each face as 1,2,3,4,5 and 6.

Step 2: Throw it once. What number can you obtain on top? Write down all the possible results when a die is thrown once.

Step 3: Throw it again. What number can you obtain on top? Write down all the possible results when a die is twice.

Step 3:

i) Complete the table on hand out and record all possible outcomes.

| At once \ At twice | 1 | 2 | 3 | 4 | 5 | 6 |
|--------------------|---|---|---|---|---|---|
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |

ii) How many ways of getting 2 and 2?

iii) How many ways of getting 1 and 3?

Step 4: from your findings in step 3, what is the probability of getting:

- i) twos?
- ii) 1 and 3?
- iii) 2 even numbers?
- iv) Sum of 5?

f) Interpretation of results and Conclusion

1) The table below shows all possible outcomes.

| At once At twice | 1 | 2 | 3 | 4 | 5 | 6 |
|-----------------------------------|----------|----------|----------|----------|----------|----------|
| 1 | (1,1) | (1,2) | (1,3) | (1,4) | (1,5) | (1,6) |
| 2 | (2,1) | (2,2) | (2,3) | (2,4) | (2,5) | (2,6) |
| 3 | (3,1) | (3,2) | (3,3) | (3,4) | (3,5) | (3,6) |
| 4 | (4,1) | (4,2) | (4,3) | (4,4) | (4,5) | (4,6) |
| 5 | (5,1) | (5,2) | (5,3) | (5,4) | (5,5) | (5,6) |
| 6 | (6,1) | (6,2) | (6,3) | (6,4) | (6,5) | (6,6) |

From the table

- 1) a pair of 2 and 2 is obtained only once.
- 2) There is only one way of getting of getting 1 and 3.

On step 4,

- i) Since twos is obtained once, then

$$P(2,2) = \frac{1}{36}$$

- ii) Since a pair of 1 and 3 becomes once, then $P(1,3) = \frac{1}{36}$

- iii) There are 6 pairs of 2 even numbers are obtained in the table that is why

$$P(2evens) = \frac{6}{36} = \frac{1}{6}$$

- iv) There are 4 ways of obtaining the sum of 5 among 36 possible ways. Then,

$$P(sum\ of\ 5) = \frac{4}{36} = \frac{1}{9}$$

g) Additional information for teacher

Teacher should have to set clear instructions.

Teacher should manage students' attitudes when throwing cards for instance, advising them against noise and unnecessary movements.

h) Guidance on the evaluation

1. Consider an event of throwing a die once.

- i) Write down the sample space.
- ii) What is the probability of each element?

Expected result

- i) The sample space is $S = \{1, 2, 3, 4, 5, 6\}$
- ii) The probability of each element is $\frac{1}{6}$ since each element is likely to occur once in one throw.

PRACTICAL ACTIVITY 1: Addition and subtraction of second order polynomials using algebraic tiles

a) Rationale:

This practical activity is conducted when teaching the lesson on simplification of second order polynomials. It is taught in Unit 2 of S2.

In real life, engineer designing a coaster would use polynomials to model the curves, while a civil engineer would use polynomials to design roads, buildings, and other structures. Polynomials are also an essential tool in describing and predicting traffic patterns so appropriate traffic control measures, such as traffic lights, can be implemented. Economists use polynomials to model economic growth patterns, and medical researchers use them to describe the behavior of bacterial colonies.

b) Objective

Use algebra tiles to model addition or subtraction of the second order polynomials. For example,

$$i) (3x^2 + 2x - 3) + (x^2 - x + 2)$$

c) List of required materials:

Ruler, scissors, Pencil, Markers, or coloured pencils (blue, green, yellow, orange, purple, red), Piece of papers and algebraic tiles.

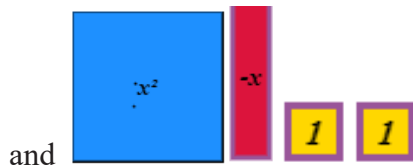
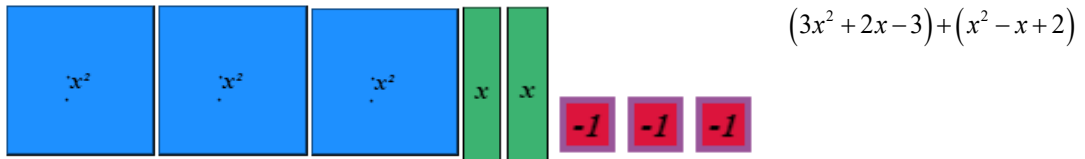
d) Illustration of the activity:



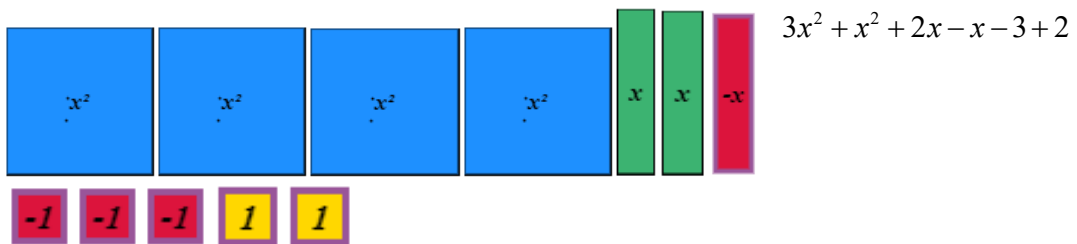
The Key above shows different-sized tiles and each tile represents a monomial. Like terms have the same shape and size, but if they are positive, they have a + (plus) sign. If they are negative, they have a - (minus) sign.

e) Procedures for adding polynomials:

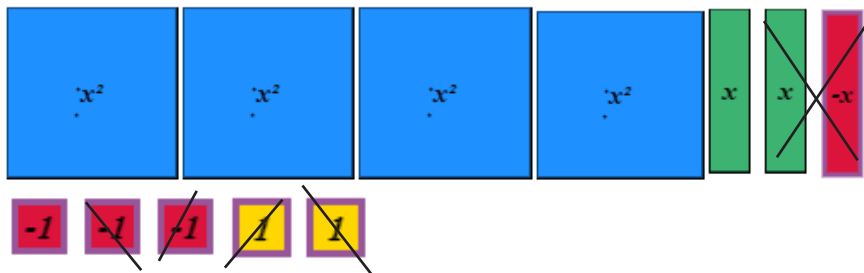
Step 1: Represent each addend with tiles.



Step 2: Rearrange tiles so that like tiles are together. Like tiles are the same size and shape



Step 3: Cancel out opposite tiles that are of the same size but have a different symbol



Step 4: Count the remaining tiles of each size and note their amount.



Step 5: Translate the tiles to a polynomial.

What have you found?

Can you do the same to simplify: $(3x^2 + 5x - 2) - (2x^2 + 2x - 1)$?

Expected answer:

Subtraction of polynomials

Number of remaining x^2 tiles is 4.

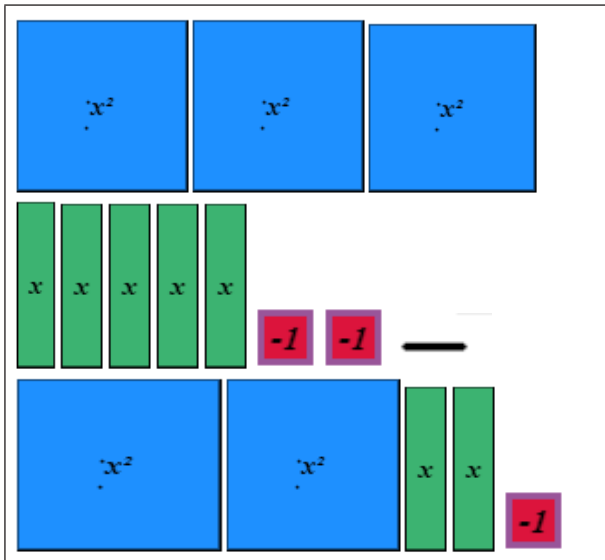
Number of x tiles remaining is 1.

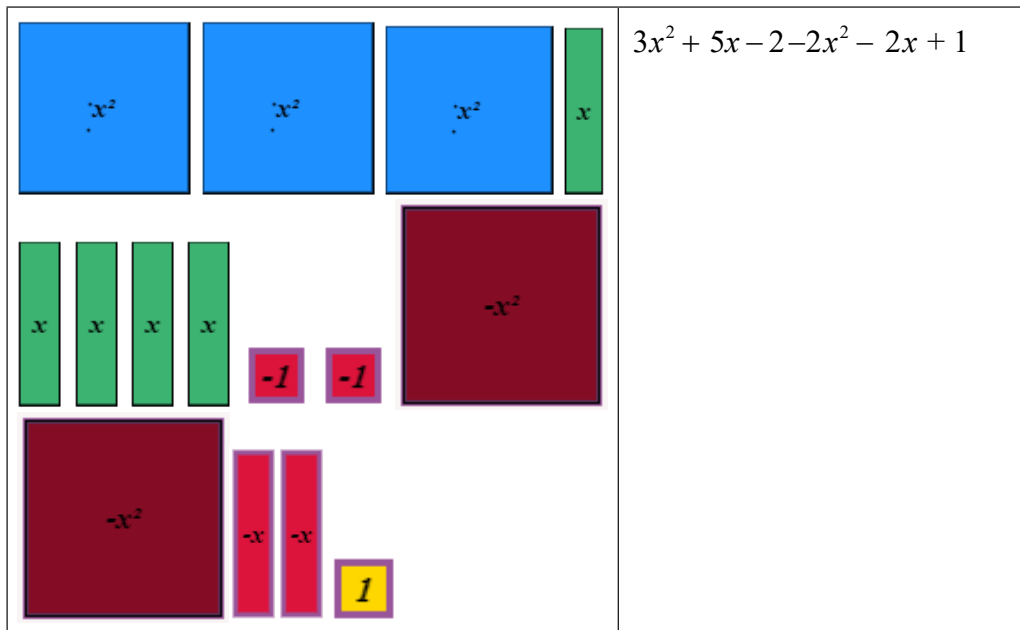
Number of -1 tiles remaining is 1.

Therefore $(3x^2 + 2x - 3) + (x^2 - x + 2) = 4x^2 + x - 1$

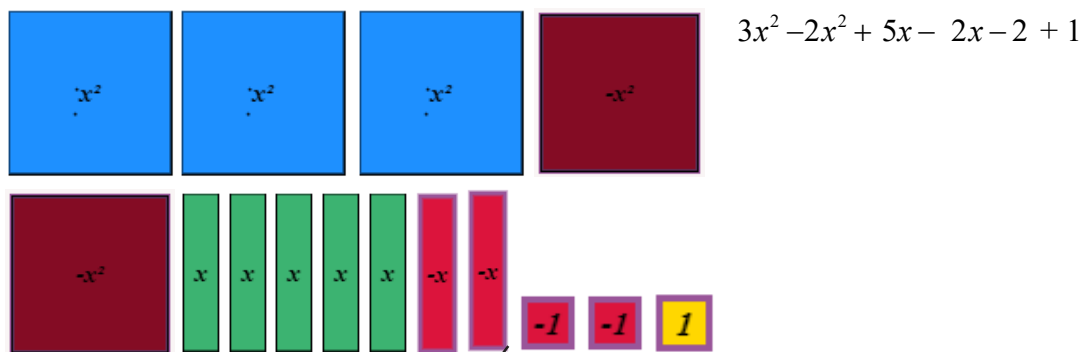
To simplify $(3x^2 + 5x - 2) - (2x^2 + 2x - 1)$, proceed as follows:

Step 1: Arrange tiles for each polynomial

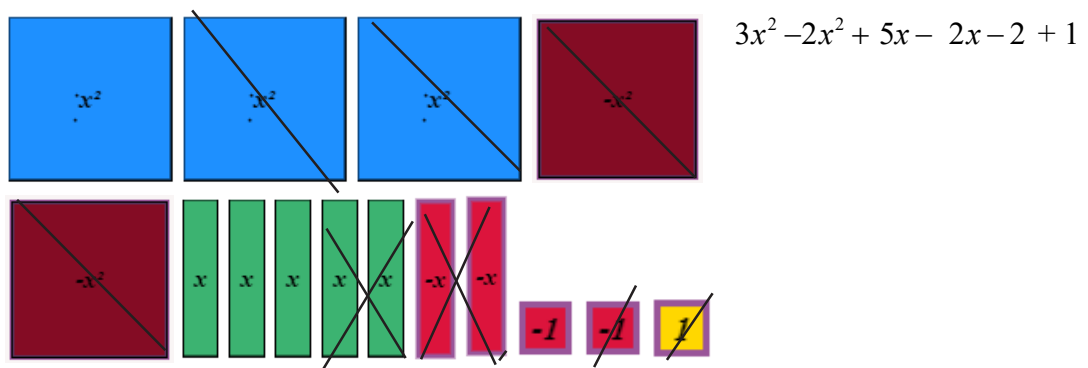
| | |
|--|-------------------------------------|
|  | $(3x^2 + 5x - 2) - (2x^2 + 2x - 1)$ |
|--|-------------------------------------|



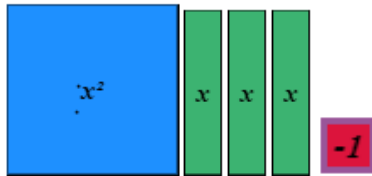
Step 2: As we have the sign -, change the color of the second polynomial



Step 3: Cancel the tiles of the same size and with different colors



Step 4: Count the remaining tiles of each type and write the corresponding polynomial



$$x^2 + 3x - 1$$

Number of remaining x^2 tiles is 4.

Number of x tiles remaining is 3.

Number of -1 tiles remaining is 1.

Therefore, $(3x^2 + 5x - 2) - (2x^2 + 2x - 1) = x^2 + 3x - 1$

f) Interpretation of results and Conclusion

What properties of addition allow you to rearrange the tiles?

Try and establish the rule of adding or subtraction second order polynomials.

g) Information for Teachers:

Teacher with learners can make local tiles from sheets of paper or manila papers: Using the ruler and pencil, divide the sheet as follows: Large squares $3\text{cm} \times 3\text{cm}$ these can be x^2 tiles, rectangles $1\text{cm} \times 3\text{cm}$ can be x tiles and small squares $1\text{cm} \times 1\text{cm}$ which are unit tiles.

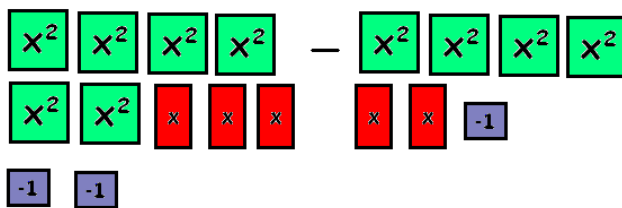
h) Guidance on the evaluation

Provides different exercises on addition and subtraction of quadratic polynomials.

For example,

Use algebra tiles to simplify: $(6x^2 + 3x - 2) - (4x^2 + 2x - 1)$

i. Model the given polynomials using algebra tiles.

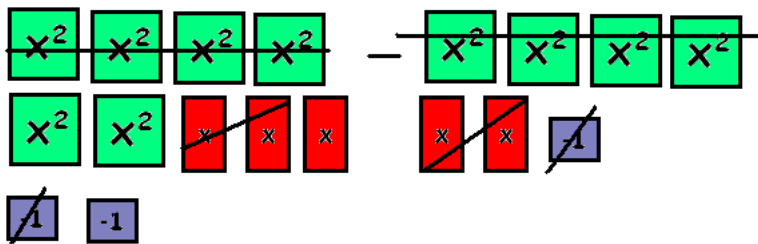


Here, we subtract two Polynomials.

ii. Cross out four x^2 tiles on the left side and do the same on the other side.

Cross out two x tiles on the left side and do the same on the other side.

Cross out one -1 tile on the left side and do the same on the other side.



iii. Remaining number x^2 tiles = 2.

Number of x tiles remaining = 1.

Number of -1 tiles remaining = 1.

So, $(6x^2 + 3x - 2) - (4x^2 + 2x - 1) = 2x^2 + 1x + 1$

PRACTICAL ACTIVITY/ EXPERIMENT 2: Quadratic identities

a) Rationale:

This experiment is conducted when teaching the lesson on quadratic identities. It is taught in Unit 2 of S2.

In real life, quadratic equations are used in many real-life situations such as calculating the areas of an enclosed space, the speed of an object, the profit and loss of a product, or curving a piece of equipment for designing.

One such real-life example is that if an object is projected, then the place where the object will reach the ground, the distance travelled by the object, and the time taken by the object to reach the peak height can all be determined using quadratic equations. A quadratic equation can be used to determine the area of a figure.

b) Objective:

To verify practically the quadratic identities:

i) $(a+b)^2 = a^2 + 2ab + b^2$

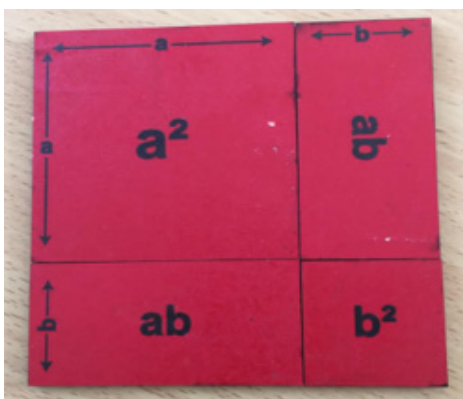
ii) $(a-b)^2 = a^2 - 2ab + b^2$

iii) $a^2 - b^2 = (a + b)(a - b)$

c) Materials required:

Cut outs for algebraic identities.

d) Illustration or diagram



e) Procedures

Step 1: Place the tile for the square of side a , its area is $a \times a = a^2$

Step 2: Place on the right the rectangle of length a and width b near the first square such that the sides a coincide. Its area is $a \times b = ab$

Step 3: Place under the first square the rectangle of length a and width b such

that sides a coincide. Its area is $a \times b = ab$

Step 4: Place the square of side b under first rectangle. Its area is $b \times b = b^2$

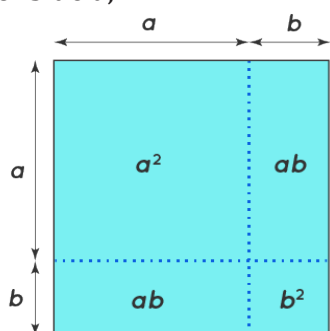
Step 5: Observe the obtained figure composed by the 4 figures. What figure is it? What are its sides? How can you find its area?

f) Results and their interpretation:

Complete the following sentences and expressions:

The area of the large square is $(a+b) \cdot (\dots + \dots)$

In the other way, the area of the large square is the sum $a^2 + \dots$ of the square of side a ,



Repeat the activity by making squares and rectangles with different values of a and b using chart paper etc. and complete the following table:

| No | a | b | a^2 | b^2 | $2ab$ | $a+b$ | $(a+b)^2$ | $a^2 + 2ab + b^2$ |
|----|-----|-----|-------|-------|-------|-------|-----------|-------------------|
| 1 | | | | | | | | |
| 2 | | | | | | | | |
| 3 | | | | | | | | |

What can you conclude about $(a+b)^2$?

Expected answers:

The algebraic expression $(a+b)^2$ is nothing but $(a+b) \times (a+b)$. This can be visualized as a square whose sides are $(a+b)$ and the area is $(a+b)^2$. The square with a side of $(a+b)$ can be visualized as four areas of a^2 , ab , ab , b^2 . The sum of these areas $a^2 + ab + ab + b^2$ gives the area of the square $(a+b)^2$. The area of the square $(a+b)^2 = a^2 + ab + ab + b^2$ proves the algebraic identity.

g) Guidance on the evaluation

In the same way, ask students to give the steps to be followed to prove:

(ii) $(a-b)^2 = a^2 - 2ab + b^2$

(iii) $a^2 - b^2 = (a+b)(a-b)$

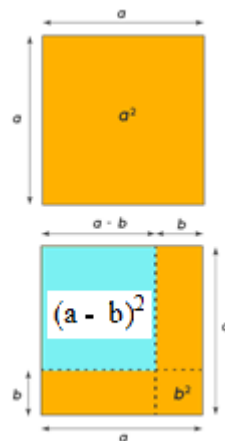
Summary of expected answer:

ii) Proof of $(a-b)^2 = a^2 - 2ab + b^2$

Let's think of $(a - b)^2$ as the area of a square with length $(a - b)$. To understand this, let's begin with a large square of area a^2 . We need to reduce the length of all sides by b , and it becomes $a - b$.

We now must remove the extra bits from a^2 to be left with $(a - b)^2$. In the figure below, $(a - b)^2$ is shown by the blue area. To get the blue square from the larger orange square, we have to subtract the vertical and horizontal strips that have the area ab . However, removing ab twice will also remove the overlapping square at the bottom right corner twice. Hence, we add b^2 .

Thus, we have $(a - b)^2 = a^2 - ab - ab + b^2$. Hence this proves the algebraic identity $(a - b)^2 = a^2 - 2ab + b^2$



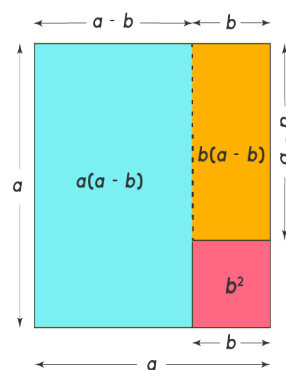
Repeat the activity by making squares and rectangles with different values of a and b using chart paper etc. and complete the following table:

| No | a | b | a^2 | b^2 | $2ab$ | $a - b$ | $(a - b)^2$ | $a^2 - 2ab + b^2$ |
|----|-----|-----|-------|-------|-------|---------|-------------|-------------------|
| 1 | | | | | | | | |
| 2 | | | | | | | | |
| 3 | | | | | | | | |

iii) Proof of $(a + b)(a - b) = a^2 - b^2$

The objective is to find the value $a^2 - b^2$, which can be taken as the difference of the area of two squares of sides a units and b units respectively. This is equal to the sum of areas of two rectangles as presented in the below figure. One rectangle has a length of a units and a breadth of $(a - b)$ units. Another rectangle is taken with a length of $(a - b)$ and a breadth of b units. Further, we take the areas of the two rectangles and sum the areas to obtain the resultant values. The respective areas of the two rectangles are $(a + b) \times a = a(a - b)$, and $(a - b) \times b = b(a - b)$. Finally, we take the sum of these areas to obtain the resultant expression.

$a(a + b) + b(a - b) = a^2 - b^2$ Re-arranging the individual squares and rectangles, we get: $(a + b)(a - b) = a^2 - b^2$



PRACTICAL ACTIVITY 3: Factorization of quadratic polynomial

a) Rationale:

This activity is conducted when teaching the factorization of expressions of the type $ax^2 + bx + c$. It is taught in Unit 2 of S2.

Quadratic polynomials are used in many real-life situations such as calculating the areas of an enclosed space, the speed of an object, the profit and loss of a product, or curving a piece of equipment for designing.

One such real-life example is that if an object is projected, then the place where the object will reach the ground, the distance travelled by the object, and the time taken by the object to reach the peak height can all be determined using quadratic equations. A quadratic equation can be used to determine the area of a figure.

b) Objective 1:

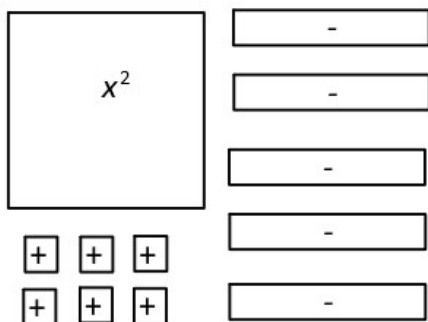
Factorize the expression $x^2 - 5x + 6$

c) Required materials:

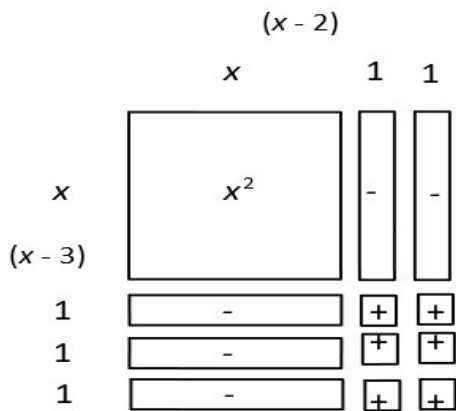
algebraic tiles.

d) Procedures and illustration

Step 1: Represent this expression using 1 x^2 tile, 5 negative x tiles, and 6 positive integer tiles.



Step 2: Arrange the tiles into a rectangle. Start with the x^2 tile in the upper left corner. Then, align the negative x tiles to the right of the square and below it, as needed. Make sure the side with length x is parallel to the side of the x^2 tile. Last, add the integer tiles in the lower right corner. Make sure each integer tile aligns with the side of a x tile with the length of 1, both above and to the left.

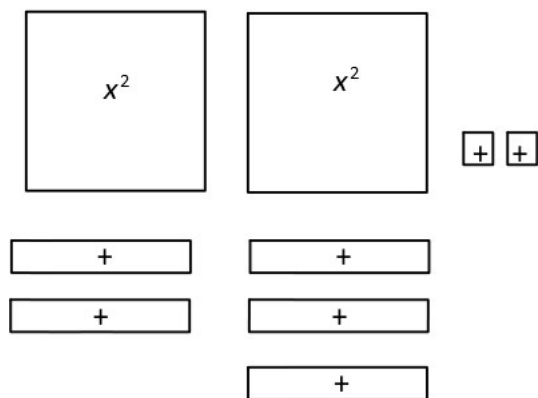


Step 3: The factors are the lengths of the sides of the rectangle. The top side of the rectangle has one x^2 tile of length x and 2 negative x tiles of length 1. Because these x tiles are negative, this is represented as $(x - 2)$. The left side has 1 tile of length x and 3 negative x tiles of length 1. This is represented as $(x - 3)$.

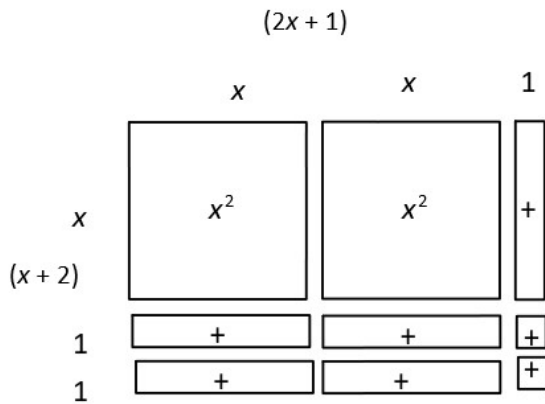
The factored expression is the product of the side lengths, or $(x - 2)(x - 3)$.

Objective 2: Factorize the expression $2x^2 + 5x + 2$

Step 1: Represent this expression using 2 x^2 tiles, 5 positive x tiles, and 2 positive integer tiles.



Step 2: Arrange the tiles into a rectangle. Note that because there are 2 x^2 tiles, there must be an x tile aligned below each.



Step 3: The factors are the lengths of the top and left sides of the rectangle. The top has $2x^2$ tiles of length x and 1 positive x tile of length 1, or $(2x+1)$. The left side has 1 x^2 tile of length x and 2 positive x tiles of length 1, or $(x+2)$.

So the factored expression is $(2x+1)(x+2)$.

Step 4: Multiply to check your answer.

$$(2x+1)(x+2) = 2x \times x + 2x \times 2 + 1 \times x + 1 \times 2 = 2x^2 + 5x + 2$$

e) Interpretation of results and conclusion

Summarize how to factor a quadratic expression of the form $ax^2 + bx + c$ by using algebraic tiles.

Expected answer:

To factorize the expression of the form $ax^2 + bx + c$, the following steps:

Step 1: Use algebra tiles to represent a trinomial of the form $ax^2 + bx + c$ where a , b , and c are integers. Include a of the x^2 tiles, b of the x tiles, and c of the integer tiles. If b or c are negative, represent them using the negative (-) tiles.

Step 2: Arrange the tiles into a rectangle.

Step 3: The factors of the trinomial are represented by the lengths of the sides of the rectangle. Remember to consider whether the x tiles are positive or negative.

Step 4: Check your answer. Multiply the factors to make sure they produce the original trinomial.

e) Guidance on the evaluation

Now using the similar process of factorisation by algebraic tiles complete the number of required tiles in the following table:

| Polynomial/Tile | x^2 | x | $-x$ | $+1$ | -1 | 1st factor | 2nd factor |
|------------------|-------|-----|------|------|------|------------|------------|
| $x^2 + 7x + 10$ | | | | | | | |
| $-2x^2 - 3x + 5$ | | | | | | | |
| $2x^2 + 10x$ | | | | | | | |
| $x^2 - 7x + 12$ | | | | | | | |

PRACTICAL ACTIVITY/EXPERIMENT 4: Exploring the Midpoint theorem

a) Rationale:

This experiment is conducted when teaching the lesson on the midpoint theorem of Thales. It is taught in unit 5 of S2.

Thales theorem is used when arranging tiles and painting. It can be useful as paintings, tiles, and many other things as such that require precision to make sure that there is a proportional way.

b) Objective

To verify that a line drawn through the midpoint of one side of a triangle and parallel to the second side bisects the third side.

c) Materials Required

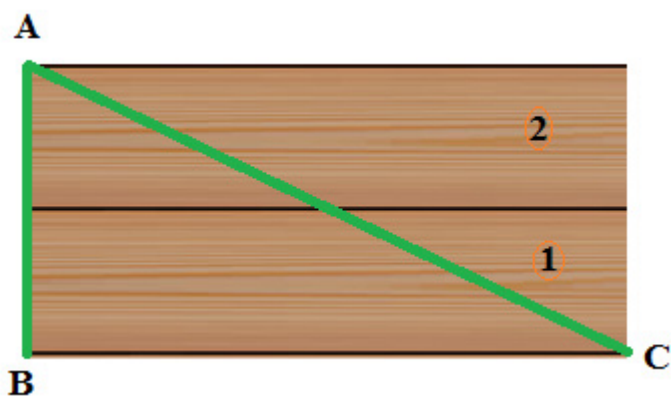
Two well prepared timbers of the same height and the same length, nails, thread, and ruler.

d) Procedures and illustration

Step 1: Paste the timber 1 and timber 2 one by another on the floor in the playground.

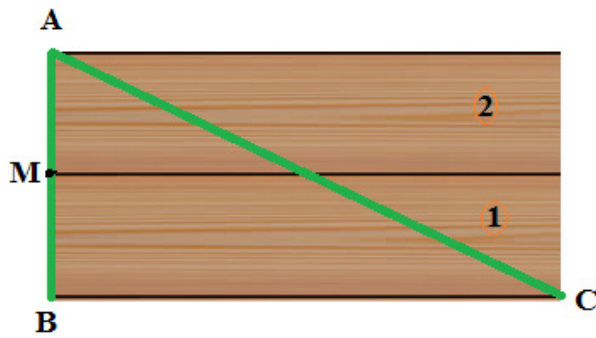
Step 2: Fix a nail A in the corner of the timber 2 and the nail B in the corner of the timber 1 situated on the left side. Such that AB is a vertical line.

Step 3: Fix one nail on the second corner of the timber 1 at the same base as B and name this nail as point C.

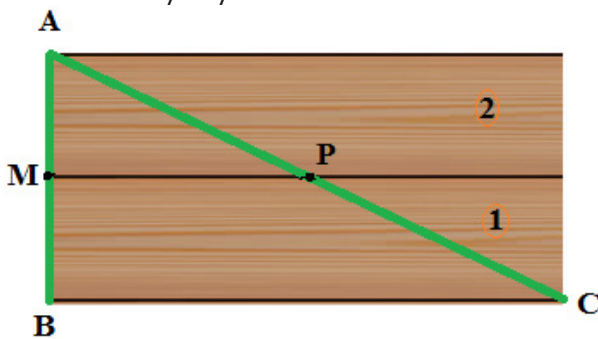


Step 4: Fix a thread joining the nail A and the nail in B. Fix another thread joining the nail A and C. How is the figure ABC? Does it have a right angle at B?

Step 5: Observe the intersecting point between the thread AB and the line of junction of the two timbers, mark it as M and measure the distance /AM/ and /MB/. Are they equal?



Step 6: Observe the intersecting point between the thread AC and the line of junction of the two timbers, mark it as P and measure the distance /AP/ and /PC/.



Are /AP/ and /PC/ equal? How are line AC and MP? Use different timbers of the same height $AM = MB$ and complete your observation in the following table:

| AM | AM | MB | AB | AP | PC | AC | $\frac{AB}{AM}$ | $\frac{AC}{AP}$ |
|------|------|------|------|------|------|------|-----------------|-----------------|
| 20cm | | | | | | | | |
| 30cm | | | | | | | | |
| 40cm | | | | | | | | |

What fraction does segment AM represent in terms of length AB?

What fraction does segment AP represent in terms of length AC?

e) Interpretation of Result and conclusion

How are lines BC and MP? What is the position of the point M on the line segment AB? What is the position of P on the line segment AC?

What is the general observation?

Expected answers:

- The figure ABC is a right-angled triangle.
- A midpoint M divides a line segment AB into two equal segments AM and MB.
- The line MP is parallel to the line BC because timbers are well prepared and have the same sizes.
- The point P is the mid-point of the line segment AC.
- The straight line through the midpoints of two sides of the triangle is parallel to the third side of the triangle.
- The length of the segment joining the midpoints of the sides of the triangle is half the length of the third side which are parallel to it.

f) Guidance on the evaluation:

Ask students to consider the same triangle ABC in which $AB = 40\text{cm}$, $AC=50$ and invite them to find the length of PC.

PRACTICAL ACTIVITY/ EXPERIMENT 5: Explore Thales' theorem.

a) Rationale:

This experiment is conducted when teaching the lesson on Thales theorem in a trapezium. It is taught in unit 5 of S2.

Thales theorem is used in tiles and painting and so on. It can be useful as paintings, tiles, and many other things as such require precision to make sure that there is in a proportional way.

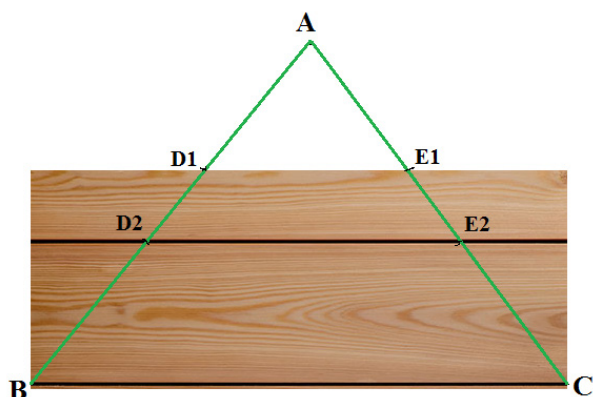
b) Objective

To verify the basic proportionality theorem, or Thales theorem that highlights that If a line is drawn parallel to one side of a triangle intersecting the other two sides, then the line divides these sides in the same ratio.

c) Materials Required

Two well prepared timbers of different heights, nails, thread, ruler.

d) Procedure and illustration.



Step 1: Place the two timbers one by another on the floor in the playground.

Step 2: Fix a nail above the two timbers on the point A.

Step 3: Fix 2 nails at the base of the first timber one at B and the other at C.

Step 4: Fix a thread joining the nail A and the nail at B. This thread passes by the upper base of the first timber at the point D2 and the upper base of the second timber at the point D1.

Step 5: Fix a thread joining the nail A and the nail at C. This thread passes by the upper base of the first timber on the point E2 and the upper base of the second timber on the point E1.

Step 6: Use the ruler and measure the lengths of the line segments AD1, D1B, AE1, E1C, AD2, D2B, AE2 and E2C, and record them in the observation table.

For each side measured on the transversal AB, what is its corresponding side on the transversal AC?

e) Recording data

| | | | | | |
|--------|--------|--------|--------|---------------------|---------------------|
| AD_1 | D_1B | AE_1 | E_1C | $\frac{AD_1}{D_1B}$ | $\frac{AE_1}{E_1C}$ |
| | | | | | |
| AD_2 | D_2B | AE_2 | E_2C | $\frac{AD_2}{D_2B}$ | $\frac{AE_2}{E_2C}$ |
| | | | | | |

f) Interpretation of Result and conclusion

How are lines BC and D1E1? How do we call lines AB and AC in respect to the parallel lines BC and D1E1?

What can you say about the values $\frac{AD_1}{D_1B}$ and $\frac{AE_1}{E_1C}$?

Can you explain $\frac{AD_2}{D_2B} = \frac{AE_2}{E_2C}$?

What is your Conclusion about the ratio of corresponding sides?

Expected answers:

The line D1E1 drawn parallel to the side BC of a triangle ABC and intersecting the other two sides (AB and AC), divides these sides in the same ratio

$$\frac{AD_1}{D_1B} = \frac{AE_1}{E_1C}$$

In the same way, $\frac{AD_2}{D_2B} = \frac{AE_2}{E_2C}$ because the line D2E2 is parallel to the side BC and it intersects the sides AB and AC. In addition, this conclusion can be gotten when we consider that the triangle AD1E1, the triangle AD2E2 and the Triangle ABC are similar.

g) information for the teacher

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

h) Guidance on the evaluation:

Ask students to consider the obtained diagram, measure the lengths of other corresponding sides and then determine the related ratio.

PRACTICAL ACTIVITY/ EXPERIMENT 6: Exploring Pythagoras theorem

a) Rationale:

This experiment is conducted when teaching the lesson on Pythagoras theorem taught in Unit 6 of S2.

Pythagoras theorem is applied to a right-angled triangle. If the lengths of any two sides of a right-angled triangle are given, then the length of the third side can be obtained.

In real life, the Pythagorean theorem is helpful in security cameras for face recognition, architects use the technique of the Pythagoras theorem for engineering and construction fields, Pythagoras theorem is applied in surveying the mountains, It is also used in navigation to find the shortest route, Painters use ladders to paint on high buildings with the help of the Pythagoras theorem.

b) Objective:

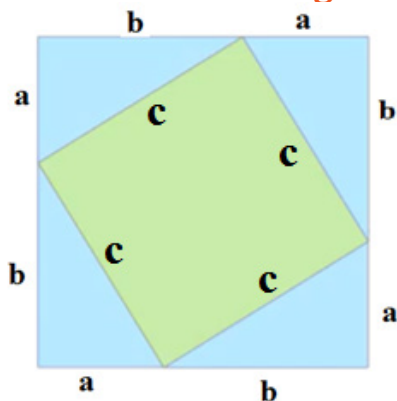
To verify practically that in a right-angled triangle, the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides. This theorem helps students to be able to understand for example how to calculate the length c of a ladder to be put on a house of length b when it stands at the distance a from the house.

c) Required materials

To conduct this activity, use the following materials:

- Cut-outs of 4 right-angled triangles with sides a , b , and c of the same sizes for which the side a and b form the right angle and the side c is the hypotenuse.
- A square of side c that is equal to the length of the hypotenuse of the triangle.
- Table or a plane surface such as the soil.

d) Illustration or diagram



e) Procedures

- Step 1.** Place the 4 right angled triangles of the same size on the table or the other plane surface in such a way that their hypotenuses c forms a space of a square of side c in side and a larger square of side $(a + b)$ on the outside.
- Step 2.** Place the square of side C in the interior part created by 4 triangles. Does the square fit in that part?
- Step 3.** Remove the internal square.
- Step 4.** Discuss how you can find the area of the square of side $a+b$ and the area of the part covered by 4 triangles. If the interior square is removed, how can we determine its area basing on the area of the large square and the area of 4 triangles?

Expected answers:

Now, we are sure that the area of the large square with side $a+b$ is the sum of the area of the interior square and the area for 4 triangles.

The area of the large square: is $(a + b)^2 = a^2 + b^2 + 2ab$

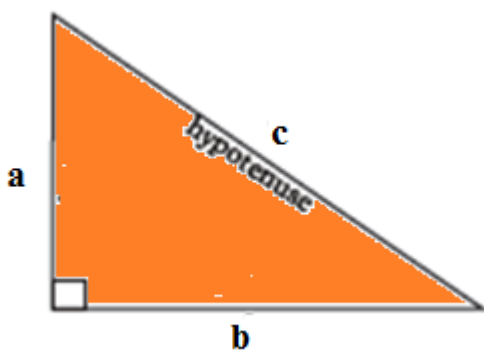
The area of the 4 triangles is: $4 \times \left(\frac{ab}{2}\right)$

The area of the interior part is the area of the square of side c . It is also the difference between the area of a large square and the area for 4 triangles.

$$c^2 = a^2 + b^2 + 2ab - 4 \times \left(\frac{ab}{2}\right) = a^2 + b^2$$

f) Interpretation of results and Conclusion

Given the right-angled triangle with side a , b and c such that c is its hypotenuse, what is the relationship between a , b and c ?



Complete: $c^2 = \dots + \dots$

What will be the length of hypotenuse if the other sides are 3cm and 4cm?

Expected answer

In a right-angled triangle with sides a , b and c where c is hypotenuse, the relationship among the sides is:

$$c^2 = a^2 + b^2$$

This is expressed as Pythagoras theorem: the square of hypotenuse is equal to the sum of the squares of the other two sides.

If $a=3\text{cm}$, $b=4\text{cm}$, we have: $c^2 = (3\text{cm})^2 + (4\text{cm})^2 = 25\text{cm}^2$

Therefore, c equals the square root of 25cm^2 which gives $c = 5\text{cm}$.

If the other sides are 3cm and 4cm , the length of the hypotenuse is 5cm .

g) Guidance on the evaluation

Ask students to use 4 right angled triangles of the same size made from cut-outs of sheets of paper such that the sides that form the right angle are $a = 5\text{cm}$, $b = 12\text{cm}$. invite them to determine the length of the hypotenuse for those triangles.

PRACTICAL ACTIVITY 7: Addition and subtraction of vectors

a) Rationale:

This activity is conducted when teaching operations of vectors. It is taught in unit 7 of S2.

In real life, vectors are used by a moving body: Navigating by air and by boat is generally done using vectors. Planes are given a vector to travel, and they use their speed to determine how far they need to go before turning or landing. Flight plans are made using a series of vectors.

b) Objective:

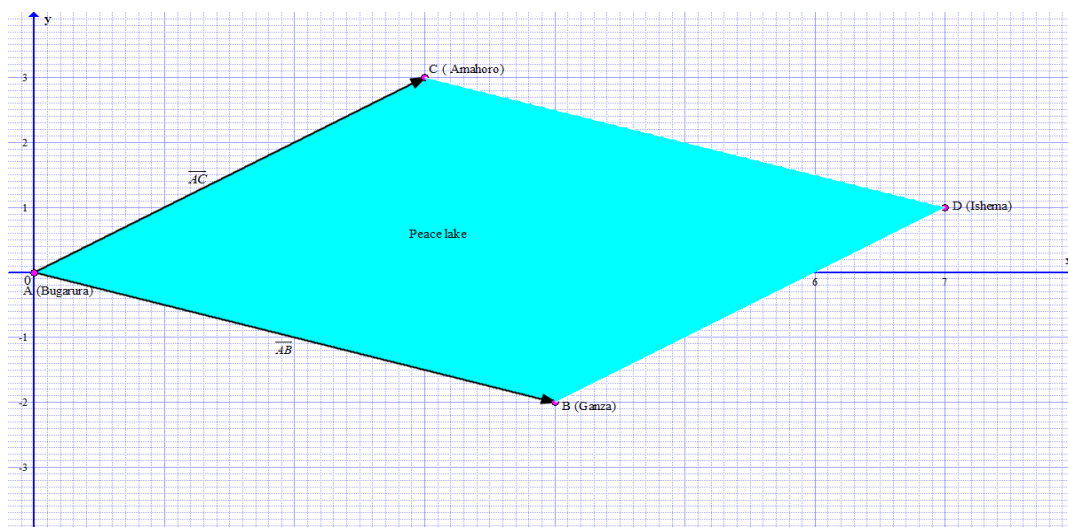
To add and subtract two consecutive vectors correctly.

c) List of required materials:

Manila paper or flipchart, marker pen, ruler, piece of paper and pen.

d) Illustration

The figure below is the illustration of different village surrounding the lake named Peace lake.



Location of Bugarura, Ganza, Amahoro and Ishema village

Isimbi moves from Bugarura to Ishema village for visiting her classmate while Muhire moves from Ganza to his native village named Amahoro.

After determining the coordinates of given village, by drawing, show the short different ways (by foot or boat), they will follow and write them as vectors. Relate your flow chart with mathematical operations. Describe the direction for each follow chart.

e) Procedures:

Step 1: Determine the coordinates of given villages

Step 2: Draw flow charts of Isimbi and write down what you notice in that flow chart.

Step 3: Describe the directions of Isimbi flow charts

Step 4: Draw flow charts of Muhire and write down what you notice in that flow chart.

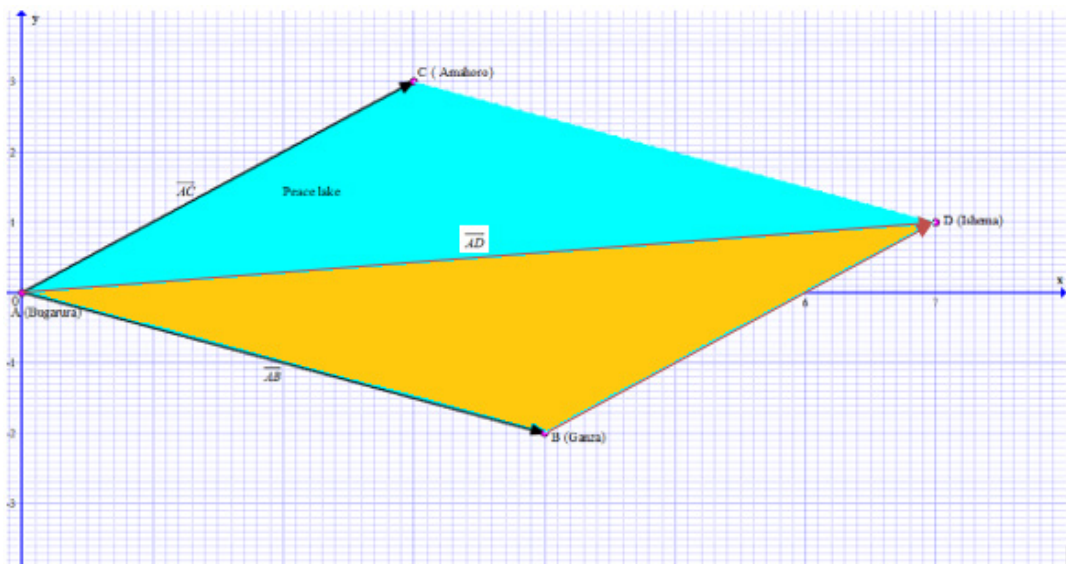
Step 5: Describe the directions of Muhire flow charts

Expected answer:

At step 1: Coordinates of village

Bugarura(0,0), Ganza(4,-2), Amahoro(3,3), Ishema(7,1).

At step 2: Flow charts of Isimbi

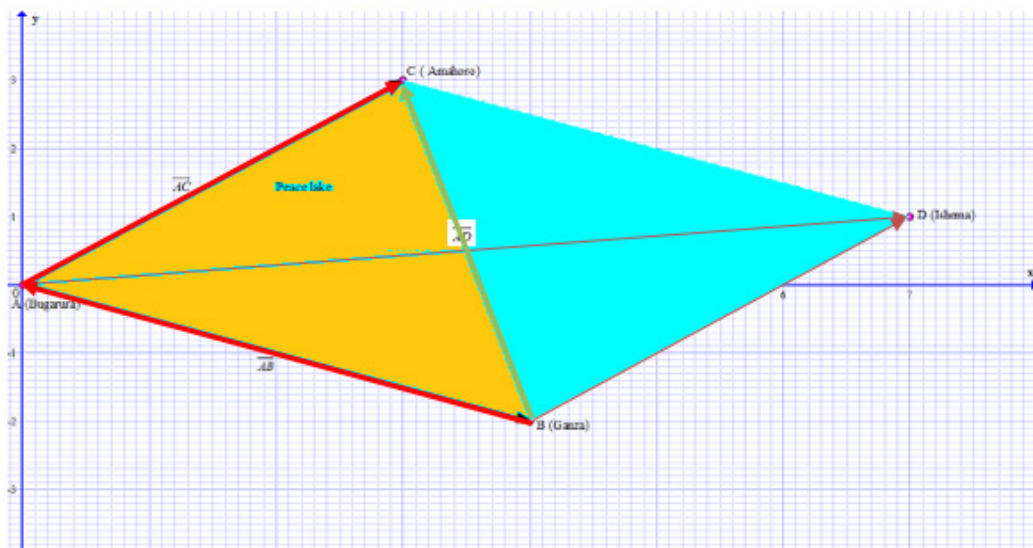


Notice: The end result of moving from A to B then from B to D is the same as going from A to D.

As the effect is the same, we can write $\vec{AB} + \vec{BD} = \vec{AD}$

At step 3: Description: \vec{AB} and \vec{BD} are in positive direction

At step 4: Flow charts of Muhire



Notice: The **end result** of moving from B to A then from A to C is the same as going from B to C.

As the effect is the same, we can write $\overrightarrow{BA} + \overrightarrow{AC} = \overrightarrow{BC}$

At step 5: Description: \overrightarrow{BA} is in negative direction while \overrightarrow{AC} is in positive direction.

f) Recording of data

Step 1: Compare \overrightarrow{BD} with \overrightarrow{AC}

Step 2: express $\overrightarrow{AB} + \overrightarrow{BD} = \overrightarrow{AD}$ in terms of \overrightarrow{AC}

Step 3: Express $\overrightarrow{BA} + \overrightarrow{AC} = \overrightarrow{BC}$ in terms of \overrightarrow{AB}

Expected answer:

Step 1: $\overrightarrow{BD} = \overrightarrow{AC}$

Step 2: $\overrightarrow{AB} + \overrightarrow{AC} = \overrightarrow{AD}$

Step 3: Express $\overrightarrow{BA} + \overrightarrow{AC} = \overrightarrow{BC} \Leftrightarrow -\overrightarrow{AB} + \overrightarrow{AC} = \overrightarrow{BC}$

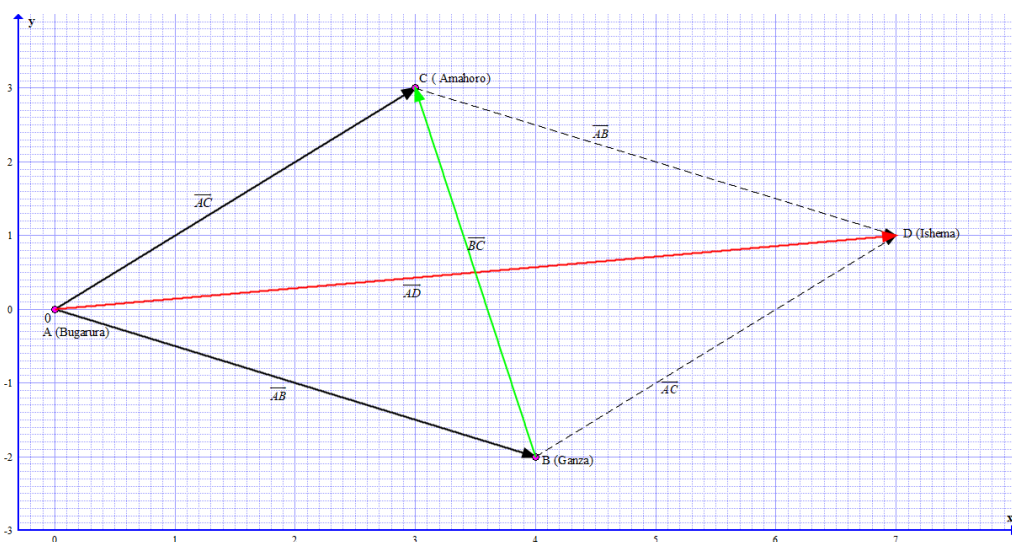
$\Leftrightarrow \overrightarrow{AC} - \overrightarrow{AB} = \overrightarrow{BC}$

g) Interpretation of results and Conclusion

- How have you drawn the resultant for sum of two vectors?
- How have you drawn the resultant for difference of two vectors?

Expected answer:

While adding or subtracting vectors, draw vectors so that their initial points coincide. Then draw lines to form a complete parallelogram. The diagonal from the initial point to the opposite vertex of the parallelogram is the **resultant**.



h) Information for Teachers:

Teacher may show the two methods of adding geometrically two consecutive vectors:

Parallelogram method and triangle method.

i) Guidance on the evaluation

students will use a manila paper, ruler and a marker to draw either a triangle or a parallelogram using vectors as sides. The figures drawn should have a starting point by laying the vectors head to tail in a sequence to create the figures.

Students will add two vectors from origin of a drawn triangle to find the resultant vector.

Apply the triangle law or parallelogram law of addition and subtraction of vectors.

PRACTICAL ACTIVITY 8: Vertical projection

a) Rationale:

This activity is conducted when teaching the vertical projection as a special case of parallel projection. It is taught in unit 8 of S2.

The parallel projection is formed by extending parallel lines from each vertex on the object until they intersect the plane of the screen. The point of intersection of the line and the screen is the projection of vertex or image of the given point. Parallel projections are used by architects and engineers for creating working drawing of the object, for complete representations that require two or more views of an object using different plane.

b) Objective:

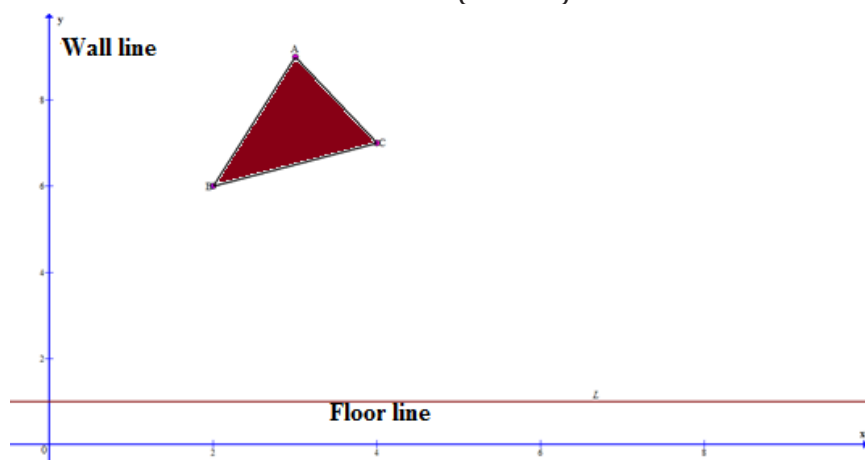
Construct the image of a geometric object painted on a wall under a vertical projection on a floor line then discuss and deduce properties of a parallel projection.

c) List of required materials:

Manila paper, a triangular object, marker, ruler and set square.

d) Illustration of the activity:

In this activity, you will project vertically the vertices of the triangle on the floor line in the direction of a wall line (corner) as illustrated below:



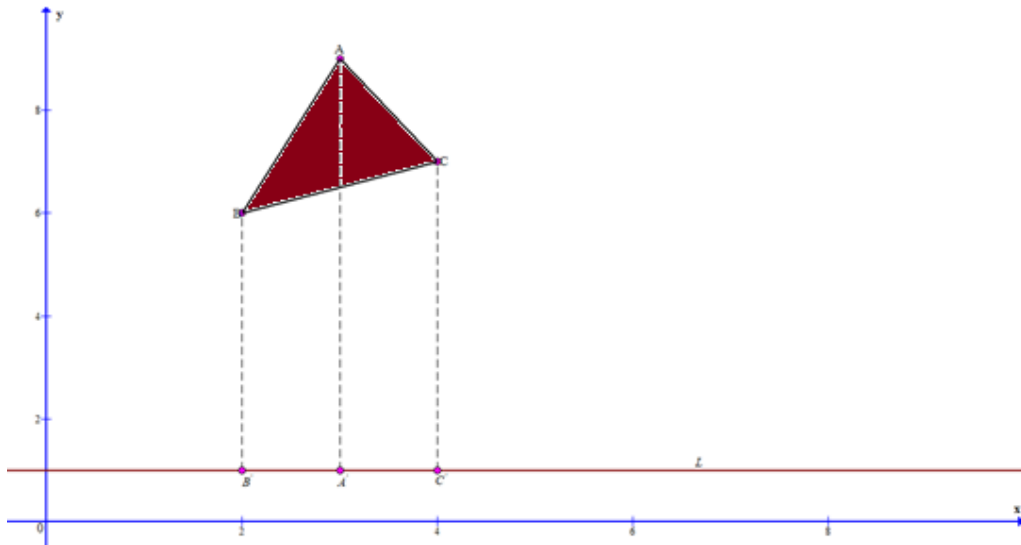
Triangle on the wall to be projected vertically on the floor line

e) Procedure:

Step 1: Hang a triangular object whose vertices are A, B and C on the wall.

Step 2: Using a ruler and a set square, draw a straight vertical line from each vertex to the floor line. You can use a small stone fixed on a thread to and put the thread at each vertex of the triangular object.

Step 3: Mark the point A' as the intersecting point of the vertical from A and the floor line. Mark the point B' as the intersecting point of the vertical from B and the floor line. Mark the point C' as the intersecting point of the vertical from C and the floor line.

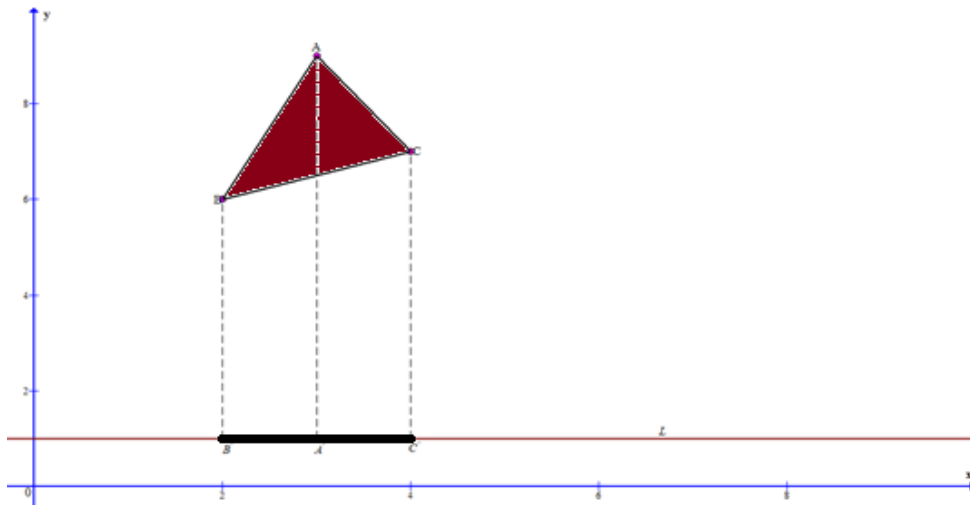


Step 4: Join the points A' and B' , B' and C' , A' and C' . what is the figure do you find? Compare the triangular object ABC and the new figure image $A'B'C'$ and discuss the properties of the obtained figure.

Expected solution:

- This figure shows a one to one mapping where A' is the image of A , B' is the image of B and C' is the image of C
- The shape of image of **triangle** ABC is a **line segment**.

f) Interpretation of results and Conclusion



How and where triangle ABC has been projected?

What was the result?

What can be the image of a line segment? What can be the image of a line? What is the image of a line segment located on the floor line? What is the image of the line segment **MP** parallel to the wall line?

Expected answers:

- The triangle has been projected by the vertical lines along the straight-line L .
- The image is a line segment.
- The triangle is projected by the vertical lines to form the image along the straight line.
- The vertical projection on one line, all images are formed on that line.
- A point on the line is mapped onto itself under vertical projection on the same line. Such a point is said to be invariant.
- Invariant points are those points which lie exactly on the line of projection under vertical projection.
- If a line segment, say MP to be projected is parallel to the direction of the projection, then the two points have the same image $M' = P'$.

g) Information for Teachers:

This activity helps students to develop knowledge and skills related to vertical projection.

Guide learners in other type of parallel projection not necessarily vertical.

PRACTICAL ACTIVITY 9: Image of object under central symmetry and properties of central symmetry

a) Rationale:

This practical activity is conducted when teaching the central symmetry and related properties. It is taught in unit 9 of S2.

Central symmetry is defined as an identical and a balanced linear similarity, which is found in two halves of an object i.e. one-half, is the linear mirror image of the other half. Symmetry is something that we observe in many places in our daily lives without even noticing it. It is easily noticeable in various arts, buildings, and monuments. So, we can easily explore the use of symmetry in real life.

b) Objective:

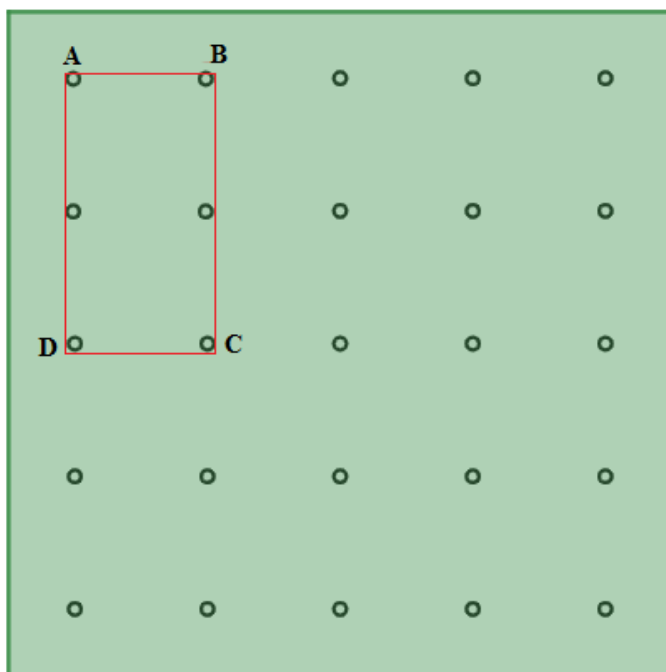
Construct the image of a given object under central symmetry. Compare the image to the initial object, then discuss and deduce the applied properties.

c) List of required materials:

Geoboard whose distance between 2 consecutive pins is 5cm, rubber bands, ruler.

d) Illustration of the activity:

Using rubber bands, construct appropriately the image of the rectangle ABCD, on geoboard under the central symmetry of centre C.



e) Procedure:

Step 1: Find the image A' of A: Join point A to C. Extend line AC to A' such that $AC = CA'$

Step 2: Similarly find the image B' of B: join point B to C and extend BC to B' such that $BC = CB'$

Step 3: Repeat procedure 1 for points C and D in order to locate images C' and D' .

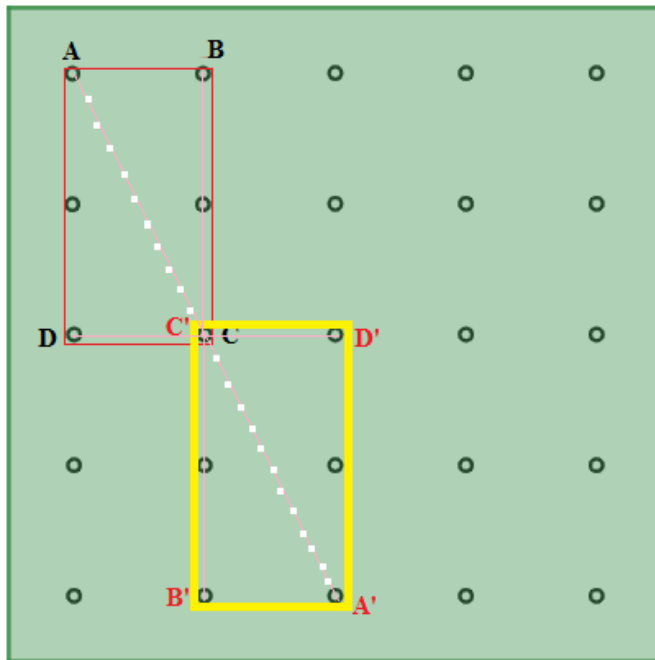
Step 4: Use a rubber band to join the points $A'B'C'D'$ in that order to obtain a closed shape.

Step 5: Describe figure $A'B'C'D'$ formed in relation to figure ABCD.

Step 6: Compare the sides and areas of ABCD with the ones of $A'B'C'D'$.

Expected answer:

At step 1-4:



At step 5: The Figure $A'B'C'D'$ is a rectangle identical to figure ABCD.

The rectangle $A'B'C'D'$ is an inverted version of ABCD.

At step 6: The two figures have same shape,

The length of side AB equals the length of side $A'B'$.

$$/AB/ = /A'B'/$$

It is the same as side AD and $A'D'$.

$$/AD/ = /A'D'/$$

The area of the rectangle ABCD is $5\text{cm} \times 10\text{cm} = 50$ square centimetres. The area

of the rectangle $A'B'C'D'$ is also $5\text{cm} \times 10\text{cm} = 50$ square centimetres. Therefore, the rectangle object and the rectangle image have the same area.

f) Interpretation of results and Conclusion

How can you name the point C according to its position from the given shape and its image?

What has been changed from this transformation?

What has not been changed from this transformation?

Expected answer:

- Point C is centre of the symmetry, it has not changed, $C = C'$
- The position of the shape has been changed (inverted).
- The shape, the size and area of the rectangle object do not change.

g) Information for Teachers:

Guide learners by emphasizing on key points of transformation by central symmetry:

- An object and its image have same shape and size.
- A point on the object and a corresponding point on the image are equidistant from the centre.
- The image of the object is inverted.
- Central symmetry is fully defined if the object and the centre are known.

h) Guidance on evaluation

Give students similar exercise with figures having different shapes.

PRACTICAL ACTIVITY 10: Image of an object under rotation

a) Rationale:

This practical activity is conducted when teaching the rotation and related properties. It is taught in unit 9 of S2.

When an object covers a certain angle around a central point, or when it rotates or spins about its axis, it is said to be exhibiting rotatory motion. Some examples of rotatory motion include the movement of hands of a clock, the motion of a spinning top, rotation of the earth and other planets, etc.

b) Objectives:

- Demonstrating the rotation in a plane.
- Construct the image of a given object under rotation, compare the image to the initial object, and then discuss and deduce the properties of rotation.

c) List of required materials:

nail, an object, a rope, Manila paper or flow chart, marker, ruler, compass, and protractor.

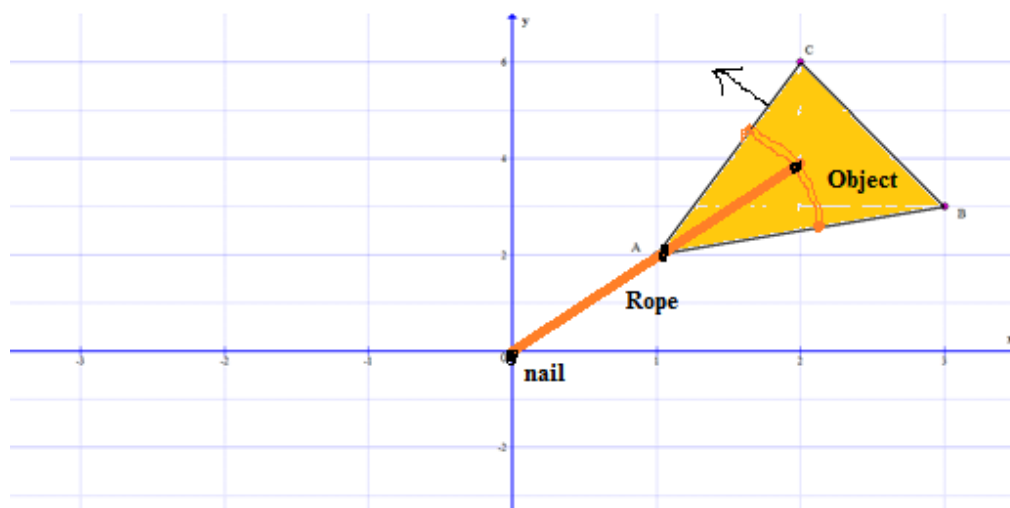
d) Procedure:

Part 1: Demonstrate the rotation of an object in a plane.

Step 1: Fix a nail at the point, mark it as O on the playground.

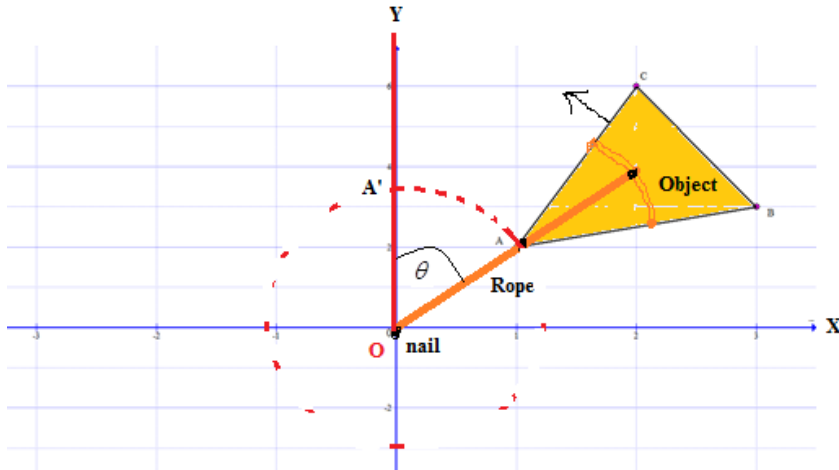
Step 2: Fix a rope to the nail.

Step 3: Attach the object to the rope.



Step 4: Draw a line from the nail and pull the object on the rope coinciding it with the line on the ground.

Step 5: Move, around the point O, the object attached to the rope in the counter clockwise direction by making an angle θ with the initial position (see the figure below).



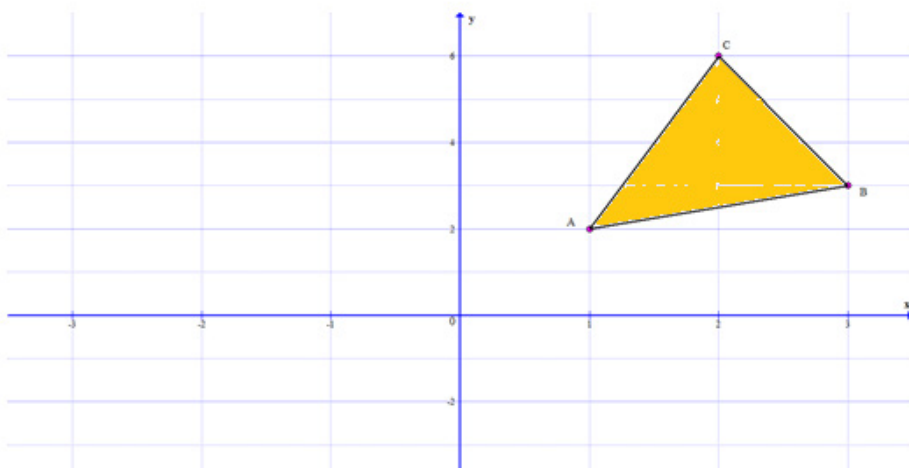
What do you observe? How do we call the angle θ ? Where is the object attached on the rope? Did the size of the object change?

Expected answer:

- The point O about which the object is rotated is called the centre of rotation
- The angle through which the object is rotated is called the angle of rotation.
- All points on the object turn through the same angle θ in the same direction.
- The shape of the object did not change; the rotation preserves shape and size.

Part 2: Construct the image of a given object under rotation

Step 1: Consider the Cartesian plane, showing a triangular object ABC; this triangle is to be rotated through a given angle, for example of 90 degrees, around origin. Join the point A to the centre O.



Step 2: Using a protractor, measure the angle 90° in anticlockwise direction and rotate the line segment OA to that angle and mark on the new line the point A' such that $OA' = OA$.

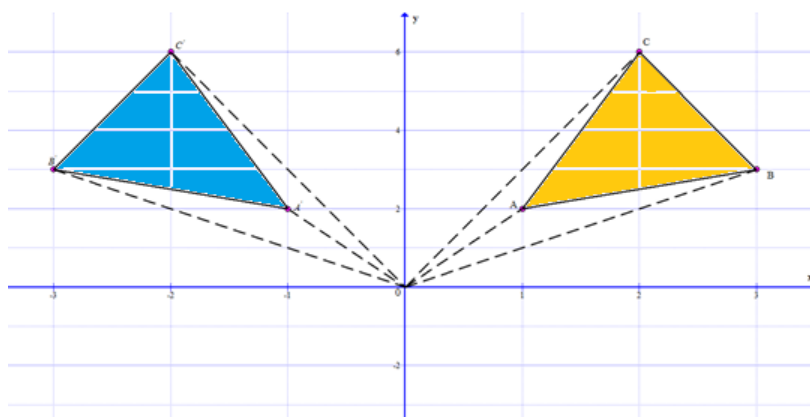
Step 3: Repeat step 1 and 2 for point B and C to get points B' and C' .

Step 4: Join the point A' , B' and C' to form the figure $A'B'C'$.

Step 5: Discuss your findings with other members of your class. What figure did you find? How are corresponding sides? How are corresponding angles?

Expected answer:

Image of the triangular object:



- The image is also a triangle.
- Corresponding sides have the same length; for example, $\angle AB = \angle A'B'$.
- Corresponding angles have the same values for example, the angle CAB equals the angle $C'A'B'$.

e) Recording of data

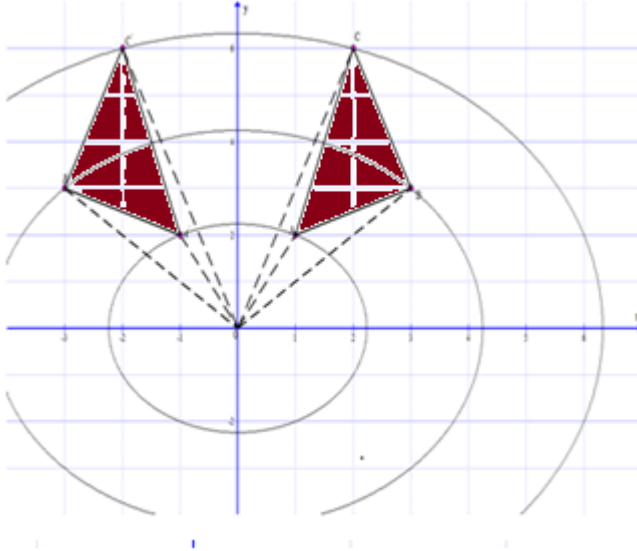
How do you check if the point A' is equidistant to A from a fixed-point O ?

How do you check if the point B' is equidistant to B from a fixed-point O ?

How do you check if the point C' is equidistant to C from a fixed-point O ?

Expected answer:

Using a compass, you draw a circle (arc) with the center at the center of rotation and a radius from the center of rotation to the point you are rotating as illustrated in the following figure.



Note: For rotation of 90° , $\text{Image}(x, y) = (-y, x)$,

The rotation looks like the way of turning an object at the given angle without changing its shape.

f) Interpretation of results and Conclusion

- What is data is needed to rotate a figure around a fixed point using a compass and protractor?
- What is the general observation?

Expected answer:

To rotate a figure around a fixed point using a compass and protractor you must have

- A centre of rotation.
- A figure to rotate.
- An angle of rotation.

The observation is the following:

- All points on the object turn through the same angle θ in the same direction.
- The shape of the object did not change; the rotation preserves shape and size.

g) Information for Teachers:

Explain that a rotation is turning a figure around a fixed point. For a rotation, you need to know the centre that the figure is rotating around, the number of degrees of the rotation, as well as the direction of the rotation. A positive number of degrees indicates anti-clockwise rotation.

h) Guidance on the evaluation

Have students draw a figure in the first quadrant on graph paper then have students flip their paper 180 degrees and draw the same figure, as if the third quadrant were the first quadrant.

Ask students to name each coordinate on the pre-image and the transformed image to determine a pattern or rule for 180-degree rotations about the origin. Repeat this exercise with 90 degree, and 270-degree rotations about the origin.

Expected answer:

For rotations $(90^\circ, 180^\circ, 270^\circ)$, through origin, we have the following transformation

| Coordinates of object | Coordinates of the image | | |
|-----------------------|---------------------------|----------------------------|---|
| | Rotation angle 90° | Rotation angle 180° | Rotation angle 270° or -90° |
| (x, y) | $(-y, x)$ | $(x, -y)$ | $(-x, -y)$ |

PRACTICAL ACTIVITY 11: Image of an object under translation and the related properties

a) Rationale:

This activity is conducted when teaching the translation of object and related properties. It is taught in unit 9 of S2.

The translation is the process of moving a shape from one place on a coordinate plane to another in the same direction. Real examples of translations are elevators, Cars moving through horizontal streets, and moving homes on a horizontal pathway.

b) Objective:

Construct the image of a given object under translation, compare the image to the initial object, and then discuss and deduce the properties of a translation.

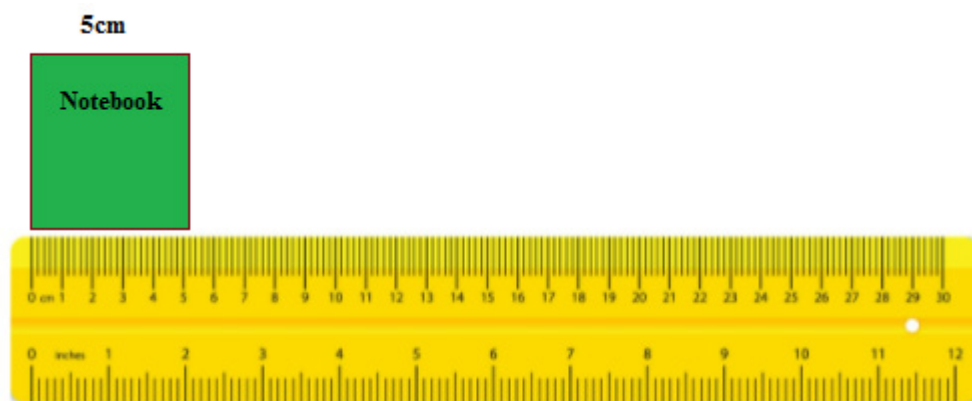
c) List of required materials:

Manila paper, graph paper, ruler, a notebook, a table, chart, marker, ruler, set square.

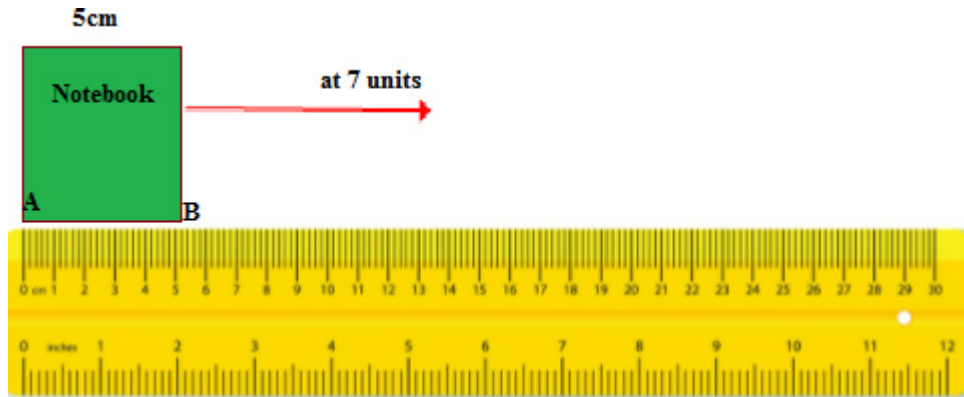
d) Procedure

Step 1: Take a ruler and put it on the table.

Step 2: Put a notebook with 5cm of width on the table such that one vertex of the notebook coincides with point 0 of the ruler.



Step 3: Push or translate the notebook at 7 units on the ruler.



What will be the position of the vertex A of the notebook?

What will be the position of the vertex B of the notebook?

Do you think that this translation on the ruler will change the size of the notebook?

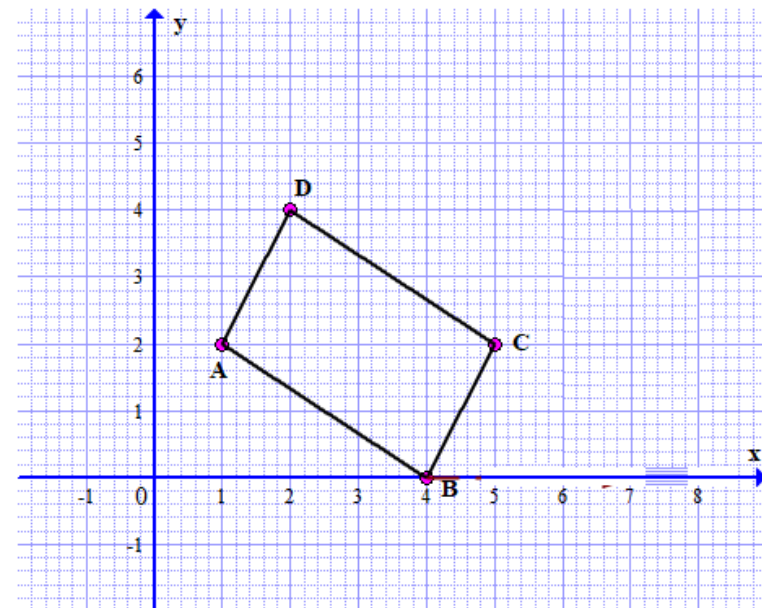
Expected answers:

After translation, the vertex A of the notebook will be at 7. The vertex B of the notebook will be at 12.

The **translation** on the ruler **does not change the size** of the notebook.

Step 4: Now, plot the vertices $A(1,2)$, $B(4,0)$, $C(5,2)$, and $D(2,4)$ on a graph paper.

Step 5: Join the vertices appropriately to get a parallelogram.



Step 6: Slide point $A(1,2)$, 3 units right then 2 units up and label it A' . In other words, translate (or push) the point A in the direction of the vector $(3,2)$ and label its new position as A' .

Step 7: Repeat step 3, for points $B(3,4)$, $C(4,0)$ and $D(5,2)$, then label their new positions as B' , C' and D' respectively.

Step 8: Join the points A' , B' , C' and D' to get the image of parallelogram $ABCD$.

Step 9: Write down your observation: What is the new figure $A'B'C'D'$?

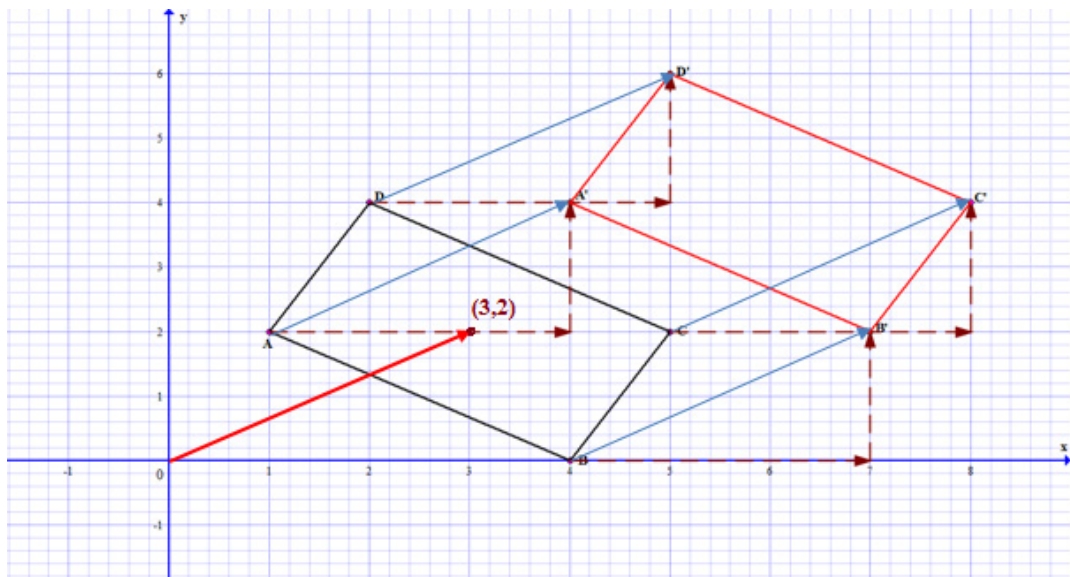
Compare the distance AD and $A'D'$. Compare the distance AB and $A'B'$.

Measure the angle DAB and the angle $D'A'B'$ and compare their values.

What can you conclude about the shape ABCD and the size of the shape $A'B'C'D'$?

Expected answer:

From step 4-8:



- The new figure $A'B'C'D'$ is a parallelogram.
- The distance AD equals the distance $A'D'$ and the distance AB equals the distance $A'B'$.
- The angle DAB and the angle $D'A'B'$ have the same value.

Therefore, parallelogram ABCD was translated in the direction of the vector $(3,2)$ but its size is the same as the size of the parallelogram $A'B'C'D'$.

e) Interpretation of results and Conclusion

- i) What do you notice about the position of the image in the 2 cases (notebook and parallelogram)?
- ii) What can you say about the size of the object and the size of the image?

- iii) What would be the coordinates of the given parallelogram if you were asked to move it 3 units to the left and 2 units down?
- iv) Given that this transformation is called translation; can you establish the rule for translation of a point?

Expected answers:

- i) The image is the object that was pushed (translated) in a given direction.
- ii) After translation of an object, the size of the image is the same as the size of the object.
- iii) If we were asked to move the parallelogram 3 units to the left and 2 units down, the direction vector is $(-3, -2)$, the coordinates of the vertices of the image for the given parallelogram are:

$$A'(1-3, 2-2) = A'(-2, 0) \quad B'(3-3, 4-2) = B'(0, 2)$$

$$C'(4-3, 0-2) = C'(1, -2) \quad D'(5-3, 2-2) = D'(2, 0)$$

- iv) The coordinates of the image vary in the following way, when we have the point P with coordinate (x, y) , to be translated in the direction of the vector $\vec{V} = (a, b)$, the coordinate of P' image of P is $P'(x + a, y + b)$.

f) Information for Teacher:

In geometry, translation means moving without rotating and without re-sizing of the object. Students may benefit from describing a translation as sliding a shape.

To translate a shape, every point on the shape must move the same distance in the same direction.

Teach students about translations by showing them a shape on a graph, and drawing a translation, such as $x+3, y+2$. Have students trace the distance each point moved horizontally and vertically and ask if they noticed a pattern. Students should notice that all the points moved specified three units right and two units up.

PRACTICAL ACTIVITY 12: Image of a given object under a rotation after a translation

a) Rationale:

This activity is conducted when teaching the composite transformation specifically the rotation after a translation of a geometric object. It is taught in unit 9 for S2.

In real life, transformations are movements through space. Consider actions as diverse as: walking and running; the factory machine that makes a thousand music CDs it rotates, moves forward etc; the opening and closing of an artificial heart valve; the movement of a robotic; artistic perspective; and the combined twists and turns of a roller-coaster. This movements are made of the combination of transformations.

b) Objective:

Construct the images of a given object under 2 successive transformations: translation and rotation.

c) List of required materials:

Manila paper or graph paper, marker, ruler, Set Square, protractor, and pair of compasses.

d) Illustration of the activity:

In a Cartesian plane plotted on the graph paper, construct a trapezium whose vertices are $A(1,1)$, $B(3,1)$, $C(4,3)$, and $D(1,3)$. Find image A', B', C', D' by translation vector $(-2,1)$, and then rotate A', B', C', D' to A'', B'', C'', D'' with a rotation angle of 90 degrees.

e) Procedure

Step 1. Plot the vertices $A(1,1)$, $B(3,1)$, $C(4,3)$, and $D(1,3)$.

Step 2: Join the vertices appropriately to get a trapezium

Step 3: Slide point $A(1,1)$, two units backward then one unit up and label it A'

Step 4: Repeat step 3, for points $B(3,1)$, $C(4,3)$, and $D(1,3)$, then label them B', C' and D' respectively.

Step 5: Join the points A', B', C' and D' to get the image of trapezium $ABCD$.

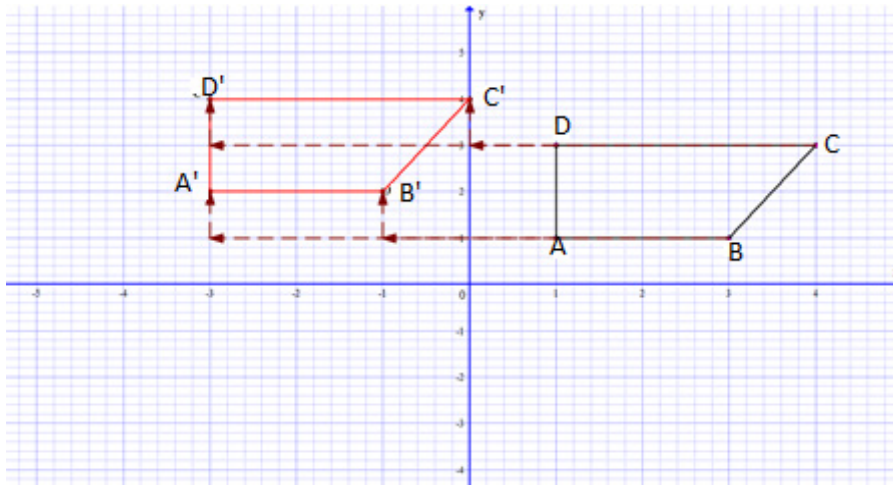
Step 6: Rotate point A' around origin through an angle of 90° to get point A'' image of A' .

Step 7: Repeat step 6 for point B', C' and D' . Join the point A'', B'', C'' and D'' to form the trapezium $A''B''C''D''$.

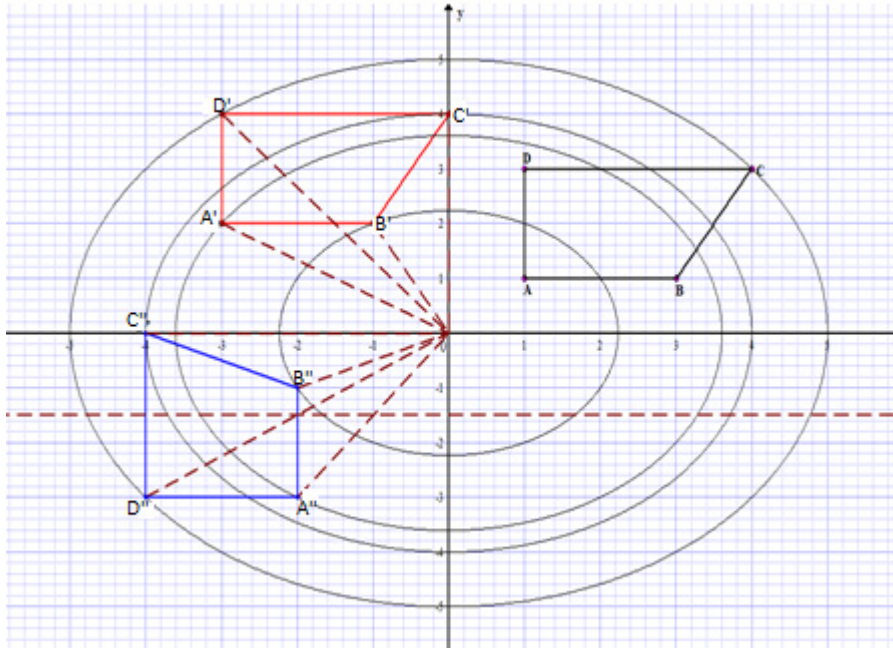
Step 8: Discuss your findings with other members of your class.

Expected solution:

Step1-5:



Step 6-7:



f) Interpretation of results and Conclusion

What can you say about the size of the last trapezium? Is it the same as the size of trapezium object?

Expected answer:

The translation did not change the size of the object. In addition, the rotation did not change the size of the obtained image. This means that the composite of the rotation and the translation did not change the size of the trapezium object.

g) information for teachers

To obtain an image of a given object under a rotation, first translate each point of the given object. then, rotate the image of the translated object on a given angle to form the transformed image.

h) Guidance on evaluation

Provide different exercises related to composite transformation.

PRACTICAL ACTIVITY 13: Frequency distribution table and a related accurate pie chart for grouped data

a) Rationale:

This activity is conducted when learning topics related to frequency distribution table and pie chart for grouped data. It is taught in unit 10 of S2.

In real life, when data are presented on a bar graph, it helps people to easily see the observation with higher frequency (more repeated) and the observation with lower frequency (less repeated).

b) Objective:

To group data in classes, make a frequency distribution table, construct a related accurate pie chart, and label it appropriately.

c) List of required materials:

Manila paper or flipchart, marker pen, Ruler, protractor, pair of compasses and calculator.

d) Illustration of the activity

Consider the following data distribution of students' results in an international test of Mathematics out of 100.

| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|
| 56 | 67 | 56 | 55 | 61 | 51 | 52 | 68 | 66 | 59 |
| 57 | 60 | 46 | 58 | 63 | 52 | 63 | 50 | 64 | 52 |
| 58 | 53 | 62 | 63 | 47 | 63 | 62 | 68 | 49 | 66 |
| 55 | 46 | 58 | 45 | 48 | 52 | 55 | 45 | 61 | 65 |

Distribute data in 5 classes and:

1. Make a frequency distribution table and ensure that all the entries are considered.
2. If each class was to be represented by a sector of an angle, calculate the degree of the sectors representing each class.
3. Construct an accurate pie chart, label it appropriately and show the percentage of candidates in each class of marks.
4. To construct a pie chart, what other fact did you require?

e) Procedures:

If data is collected in numbers as given, follow these steps to make a pie chart.

Step 1: Given that the number of classes is 5, find the range and deduce the class width.

Step 2: Find each class and the class frequency.

Step 3: Make a frequency distribution table.

Step 4: Find the angle related to frequency distribution for each class.

Step 5: Construct accurately related pie chart and label it.

Step 6: Determine the percentage for each class frequency when needed.

Expected answer:

Step 1-3: Frequency distribution

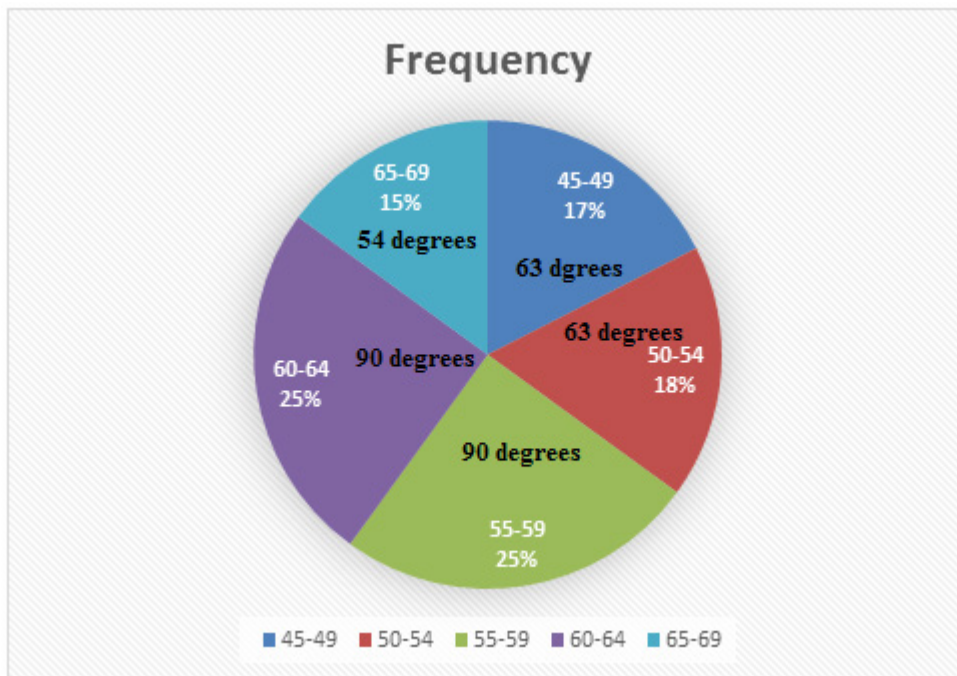
| Class | Tally | Frequency | Cumulative frequency | Boundary | Percentage |
|-------|---------|-----------|----------------------|-----------|------------|
| 45-49 | ### | 7 | 7 | 44.5-49.5 | 17.5 |
| 50-54 | ### | 7 | 14 | 49.5-54.5 | 17.5 |
| 55-59 | ### ### | 10 | 24 | 54.5-59.5 | 25 |
| 60-64 | ### ### | 10 | 34 | 59.5-64.5 | 25 |
| 65-69 | ### | 6 | 40 | 64.5-69.5 | 15 |

Step 4:

| Class | Frequency | Fraction | Angle at the centre of the pie chart |
|-------|-----------|-----------------|--------------------------------------|
| 45-49 | 7 | $\frac{7}{40}$ | $\frac{7}{40} \times 360 = 630$ |
| 50-54 | 7 | $\frac{7}{40}$ | $\frac{7}{40} \times 360 = 630$ |
| 55-59 | 10 | $\frac{10}{40}$ | $\frac{10}{40} \times 360 = 900$ |
| 60-64 | 10 | $\frac{10}{40}$ | $\frac{10}{40} \times 360 = 900$ |
| 65-69 | 6 | $\frac{6}{40}$ | $\frac{6}{40} \times 360 = 540$ |

f) Interpretation of results and Conclusion

Basing on your findings, draw a related pie chart and label it



g) information for the teacher

When given collected data, first form a frequency distribution table based on interval given, if not given you decide it. Group the data using materials described in (b) above, construct a related accurate pie chart for the grouped data.

h) Guidance on evaluation

Ask students to use a tape measure and measure the height for every student in their classroom. Group data in 6 classes and draw a pie chart illustrating representing all information about the number of students in each class.

PRACTICAL ACTIVITY 14: Exploring the likelihood of a situation to happen

a) Rationale:

This activity is conducted when teaching the likelihood of a situation to happen. It is taught in unit 11 of S2.

In real life, Probability is used in all types of areas in real life including weather forecasting, sports betting, investing, and more. Probability is used by weather forecasters to assess how likely it is that there will be rain, snow, clouds, etc. on a given day in a certain area. Probability is heavily used by sports betting companies to determine the odds they should set for certain teams to win certain games. Based on the probability, the company would offer a higher pay-out for people who bet on team B to win since it's highly unlikely that team B will actually win.

b) Objective:

To Explore the likelihood of a situation to happen.

c) List of required materials:

A tiled surface or a manila paper with 10 squares of different colours as follow: blue: 3, red: 4, pink: 2 and brown: 1, bottle tops.

d) Procedure

Step 1: Use a tiled surface, Or a Manilla paper with 10 drawn squares of different colours as shown below:



Step 2: Throw a bottle top and guess the probability of landing on one of the given colors below:

- a) blue
- b) pink
- c) blue or pink
- d) not brown

Step 3: Arrange the obtained probabilities on the probability scale



Step 4: Refer to your findings and explain when we have impossible event, certain event, event with even chance, most likely event, and less likely event.

Expected answers:

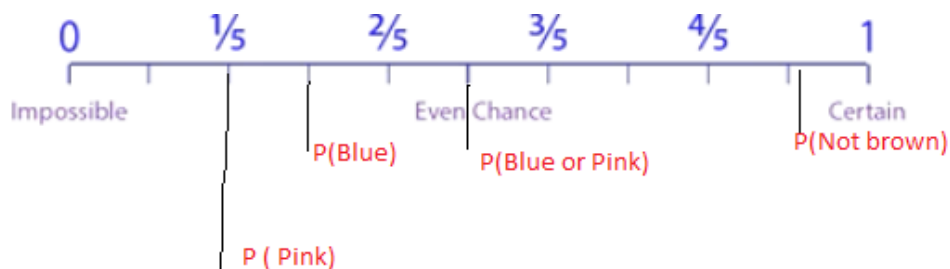


Blue Probability $P(\text{Blue}) = \frac{3}{10}$

Pink $P(\text{Pink}) = \frac{2}{10} = \frac{1}{5}$

Blue or Pink Probability $P(\text{Blue or Pink}) = \frac{5}{10} = \frac{1}{2}$

Not brown Probability $P(\text{Not brown}) = \frac{9}{10}$



e) Interpretation of results and Conclusion

From the given example explain: Certain event, most likely event, less likely event, and impossible event.

As seen from the activity, the probability that the bottle top lands on a not brown colour is a most likely event, Probability (blue or Pink colour) has even chance while Probability to get Blue colour is a less likely event. Probability of getting a green colour is an Impossible event.

f) Information for Teachers:

Teacher may give different examples of events to help students understand likelihood of events.

PRACTICAL ACTIVITY 15: Probability of the occurrence of a real-life situation using Venn diagrams

a) Rationale:

This practical activity is conducted when teaching the determination of probability Using Venn diagrams. It is taught in unit 11 of S2.

Probability is used in all types of areas in real life including weather forecasting, sports betting, investing, and more. Political forecasters use probability to predict the chances that certain candidates will win various elections. For example, a forecaster might say that candidate A has a 60% chance of winning, candidate B has a 20% chance of winning, candidate C has a 10% chance of winning, etc. to give voters an idea of how likely it is that each candidate will win.

b) Objective:

Determining probability of the occurrence of a real-life situation using Venn diagrams.

c) List of required materials

Manila paper or flip chart, markers and a pair of compasses.

d) Illustrating problem:

Use a three-way Venn diagram to solve the following problem on probability.

In a survey of 50 people about which Hotels they patronize among Hilltop, Serena, and Lemigo. We find that 15 people eat at Hilltop, 30 people eat at Serena, 19 people eat at Lemigo 8 people eat at Hilltop and Serena, 12 people eat at Hilltop and Lemigo, 7 people eat at Serena and Lemigo. 5 people eat at Hilltop, Serena, and Lemigo.

- What is the chance that a person selected at random eats only at Hilltop?
- How many eat at Hilltop and Serena, but not at Lemigo?
- How many people don't eat at any of these three hotels?
- What is the probability that a person selected at random do not eat at any of the hotels mentioned?

e) Procedure:

Step 1: Read the situational problem and record data.

Step 2: Draw a three-way Venn diagram and discuss how to complete it considering the information given in the problem.

Step 3: Start with the most specific information you have (basic regions).

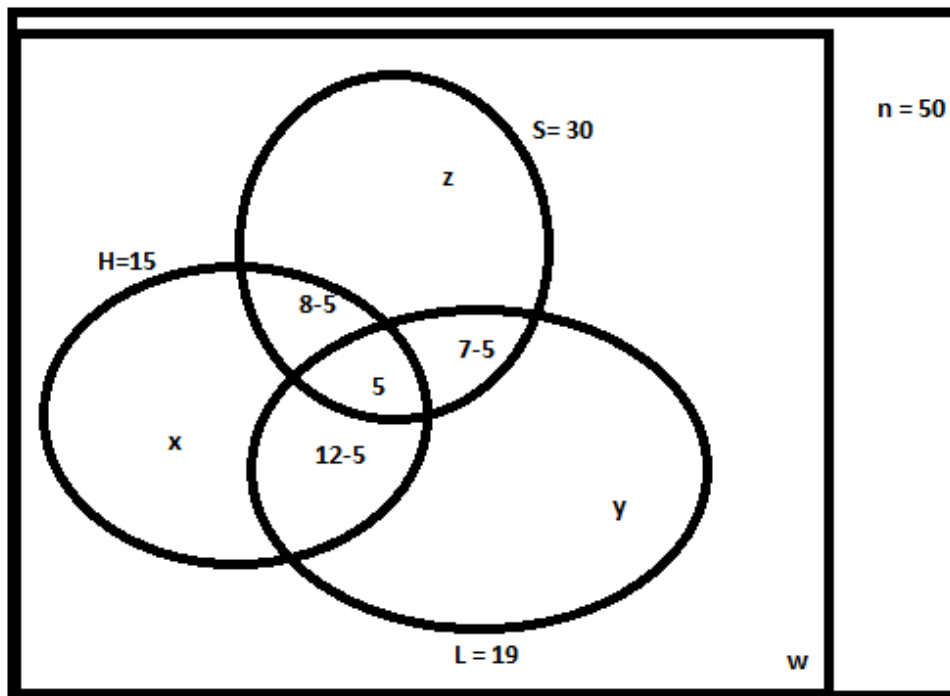
Step 4: If you can't use a piece of information yet, pass over it and try again later.

Step 5: As you interpret the information, remember:

- i) **And** implies **intersection**.
- ii) **Or** implies **Union**.
- iii) **Not** implies **complement**.

Step 5: Consider a Venn diagram of 3 circles. Let H represent Hilltop, S represent Serena and L represent Lemigo. Let x represent those who eat at Hilltop only, y represent those who eat at Lemigo only, z represents those who eat at Serena only and w represent those who don't eat at any Hotel.

Expected answer:



Step 6: Find x , y , z and w .

Step 7: For each event, find the favourable outcomes, the total outcomes and the probability requested.

Expected answer:

(a) For those who eat at Hilltop only we can find the value of x .

$$x + 12 - 5 + 5 + 8 - 5 = 15$$

$$x + 15 = 15$$

$$x = 15 - 15$$

$$x = 0.$$

No one eats at Hilltop only. And therefore, the chance that a person eats at only hilltop is zero.

(b) For those who eat at Hilltop and Serena but not Lemigo, we get $8 - 5 = 3$ people.

(c) For those who don't eat at any hotel, we can find the value of w .

$$x + 8 - 5 + 5 + 12 - 5 + 7 - 5 + y + z + w = 50$$

$$\text{But } y = 19 - (12 - 5 + 5 + 7 - 5) = 19 - 14 \text{ Hence } y = 5.$$

$$\text{And } z = 30 - (7 - 5 + 5 + 8 - 5) = 20$$

$$\text{So, } 0 + 3 + 5 + 7 + 2 + 5 + 20 + w = 50$$

$$w = 50 - 42$$

$$w = 8$$

Those who don't eat in any Hotel are 8 people.

(d) The chance that a person selected at random does not eat at any of the hotels mentioned is

$$\frac{\text{Number of people who do not eat at any restaurant}}{\text{Total number of people in the survey}} = \frac{8}{50}$$

f) Interpretation of results and conclusion

Is it easy to determine the probability without drawing a Venn diagram?

Expected answer:

We can clearly see that without a Venn diagram, some probability situations become somehow difficult to analyse.

A Venn diagram refers to representing mathematical or logical sets pictorially as circles or closed curves within an enclosing rectangle (the universal set), common elements of the sets represented by intersections of the circles.

g) Information for teacher

Venn diagrams are useful to easily analyse a probability of the occurrence of a real-life situation.

Materials described in (b) above are used by learners individually, in pairs or groups to carry out a given activity.

h) Guidance on the evaluation

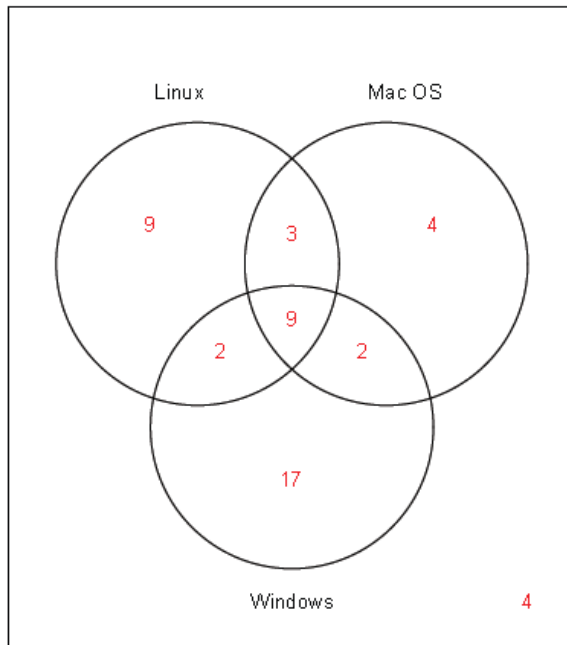
Give students another activity to be done using Venn diagram. For example:

Three of the major commercial computer operating systems are Windows, Mac OS, and Linux. A Computer Science professor selects 50 of her students and asks which of these three operating systems they use. She has notified that 30 students use Windows, 16 students use at least two of the operating systems, 9 students use all three operating systems, 18 students use Mac OS, 46 students use at least one of the operating systems, 11 students use both Windows and Linux, 11 students use both Windows and Mac OS.

- a) Use the above information to complete a three-way Venn diagram.
- b) Using the Venn diagram summarizing the distribution of operating system use previously described, what is the chance that a person selected at random
- use only windows?
 - does not use any of the operating system mentioned?
- c) How could it have been done easily? Discuss.

Expected answers:

a) Venn diagram



b) (i) $p(\text{Only Windows}) = \frac{17}{50}$.

(ii) $p(\text{No any Windows}) = \frac{4}{50}$.

The simplest way is the use of a Venn diagram to find number of different users and their probability to use the operating systems.

PRACTICAL ACTIVITY/ EXPERIMENT 16: Using a tree diagram to determine sample space, favourable outcomes, and probability that an event happens

a) Rationale:

This experiment is conducted when teaching the determination of probability using tree diagram. It is taught in unit 11 of S2. A probability tree diagram is one way to study the sample space of an experiment. They are often helpful for studying outcomes and probabilities for multistage experiments. Any situation where an “experiment” is multistage can serve as a tree diagram real life example.

b) Objective

To construct a tree diagram corresponding to the given event or situation from the real-life situation and determine the favourable outcomes, the number of all possible outcomes (sample space), and the probability for the event to happen. Event: Tossing a coin twice, tossing a coin followed by throwing a die.

c) List of required materials:

Manila paper or flip chart, sticks, coins, dice, marker, ruler, piece of paper, pens,



d) Procedures and illustration

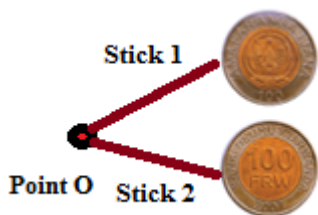
Tossing a coin twice

Step 1: Put a manila paper on a plane surface such as a table or a tiled floor,

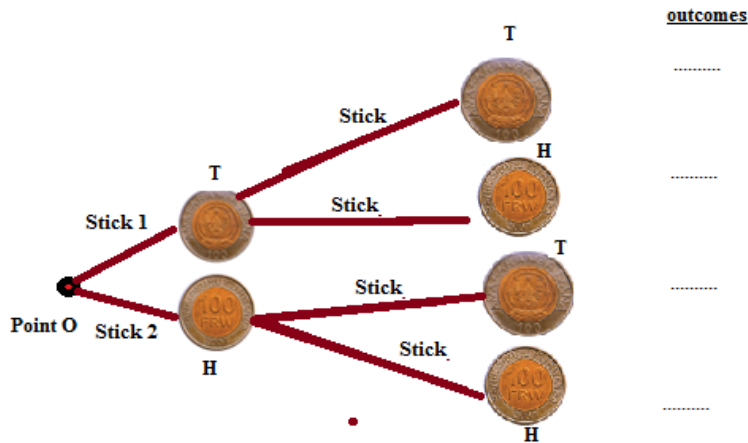
Step 2: Fix a point on a manila paper

Step 3: Toss a coin twice and write related outcomes:

i) Illustrate outcomes of the first toss using sticks. How many possible outcomes can you get?



ii) Toss the coin for the second round. How many outcomes can you get? Use stick to illustrate them on each case you got at the first round.



Step 4: After representing all possible outcomes determine the probability of the following event:

- i) Getting only heads (H)
- ii) Getting H followed by T
- iii) Getting only one H

Expected answers

All possible outcomes while tossing a coin twice $S = \{HH, HT, TH, TT\}$

i) The probability of getting only heads (H) is $p(HH) = \frac{1}{4}$

ii) The probability of getting H followed by T is $p(TH) = \frac{1}{4}$

iii) Determine the probability of getting only one H. $p(HT \text{ or } TH) = \frac{2}{4} = \frac{1}{2}$.

e) Interpretation of results and conclusion

Is it possible to find the probability without constructing a tree diagram?

Why do we call it a tree diagram?

How do you calculate the probability of an event?

$$\text{Probability} = \frac{\text{Favorable outcomes}}{\text{Total outcomes}}$$

Combination of tossing a coin and throwing a die

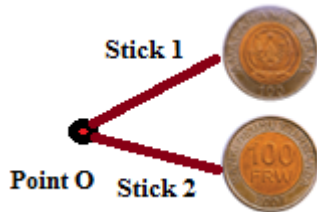
f) Procedure

Step 1: Put a manila paper on a plane surface such as a table or a tiled floor.

Step 2: Fix a point on a manila paper.

Step 3: Toss a coin once followed by throwing a dice and write related outcomes.

- i) Illustrate outcomes of tossing a coin by using sticks. How many possible outcomes can you get?



- ii) Throw a die and discuss all possible combinations of outcomes to be obtained when a coin is tossed once followed by a die thrown once.
- ii) Then use sticks of a tree diagram to illustrate all possible combinations of outcomes.
- iii) Draw a related illustration and complete the sample space.

Step 4: After representing all possible outcomes determine the probability of the following event:

- 1) i) P (Getting a head when you toss a coin once)
ii) P (getting an even number when you throw a die once)
iii) P (Getting a head and an even number when a coin is tossed once followed by a die thrown once)

Is there any relationship between the answer you got in iii) and answers gotten in i) and ii)?

- 2) P (Getting tail and odd number)
3) P (Getting Head and a counting number).

Expected answers

The possible outcomes are (H1), (H2), (H3), (H4), (H5), (H6), (T1), (T2), (T3), (T4), (T5), (T6). We obtain 12 possible number of outcomes.

- 1) (i) $P(\text{Getting a head when you toss a coin once}) = \frac{1}{2}$
ii) $P(\text{getting an even number when you throw a die once}) = \frac{3}{6} = \frac{1}{2}$
iii) $P(\text{Getting a head and an even number when a coin is tossed once followed by a die thrown once}) = \frac{3}{12} = \frac{1}{4}$

No relationship since the sample space is different in each case.

- 2) $P(\text{Getting tail and odd number}) = \frac{3}{12} = \frac{1}{4}$

$$3) P(\text{Getting Head and a counting number}) = \frac{6}{12} = \frac{1}{2}$$

g) Interpretation of results and conclusion

Was it easy to determine the probability without using a tree diagram?

Why do we call it a tree diagram?

Expected answer

It has branches and sub-branches which help to see the sequence of events and all the possible outcomes at each stage.

f) information for the teacher

Tree diagrams are useful to easily analyse a sequence of the occurrence of an event.

Materials described in (b) above are used by learners individually, in pairs or groups to carry out a given activity.

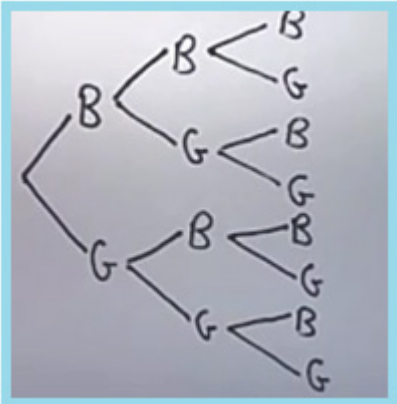
h) Guidance on evaluation

Teacher can ask student the following simple question to show them the use of tree diagram to solve problems in real life situation. For example.

1) If a family has 3 children, what is the probability that they have:

- a) Exactly one boy?
- b) Exactly 3 girls?
- c) At most two boys?

Expected answer

| The related tree digram | Sample space |
|---|--|
|  | <p>$\{BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG\}$</p> <p>It has 8 elements.</p> |

Therefore,

- a) There are 3 favourable outcomes of having exactly one boy: BGG, GBG, and GGB.

$$P(\text{Exactly one boy}) = \frac{3}{8}$$

b) There is one favourable outcome of having exactly 3 girls: GGG.

$$P(\text{Exactly 3 girls}) = \frac{1}{8}$$

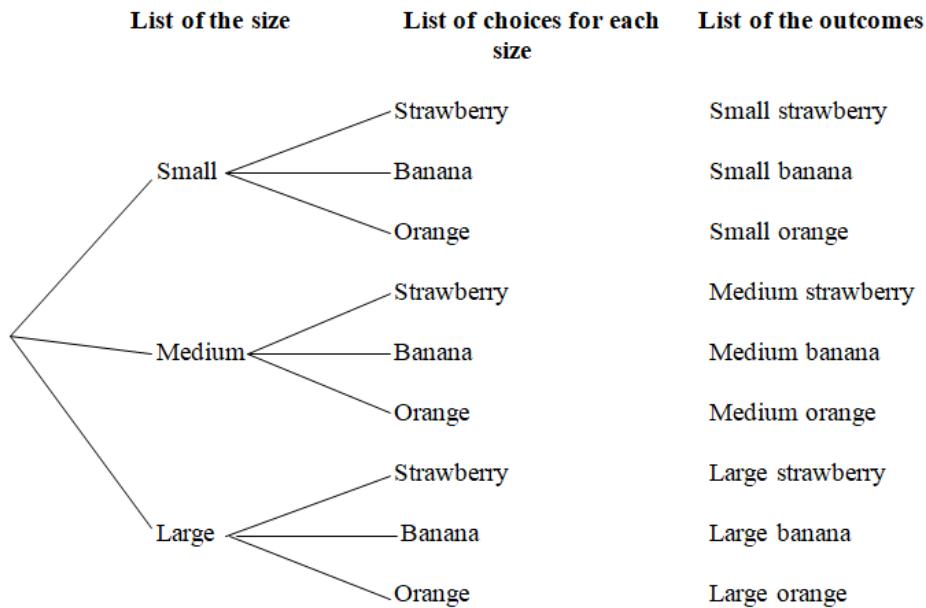
c) There are 7 favourable outcomes of having at most two boys (2 boys in maximum):

BBG, BGB, BGG, GBG, GGB, GGB, GGG

$$P(\text{Exactly one boy}) = \frac{7}{8}$$

2) You are ordering a fruit smoothie. You have your choice of a small, medium, or large smoothie, and you can include one of the following fruits: strawberries, bananas, or oranges. Make a tree diagram to find all the possible choices for smoothies.

Expected answer:



There are 9 different outcomes or possible choices for smoothies.

PRACTICAL ACTIVITY 1: Changing from base ten to base two.

a) Rationale

This practical activity is done when teaching the lesson of changing from number base 10 to any number base. It is taught in unit 2 of S3.

In real life, a number base is the number of digits or combination of digits that a system of counting uses to represent numbers. A base can be any whole number greater than 0. The most used number system is the decimal system, commonly known as base 10. Its popularity as a system of counting is most likely because we have 10 fingers.

b) Objective:

Use counters to change the number from base ten to base 2.

c) Required Materials:

12 counters, beans or stones, small boxes, and big boxes.



d) Hazard notification:

When using stones, sharpened stones can harm learners, so verify stones for each group or use beans if counters are not available.

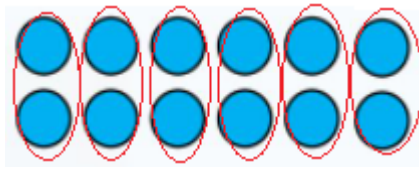
e) Source of error:

Pupils may fail to group when they do not understand instruction. Try to verify how they are grouping counters/beans.

f) Procedures and steps

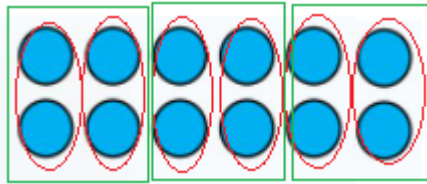
Step 1: Organize a pile of 12 counters.

Step 2: From 12 counters, organize groups of two counters

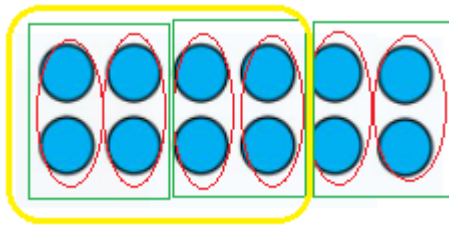


Step 3: Put a group of two containers obtained from step 3 into one small container. Record the number of counters which remain without being in a pair.

Step 4: Put a group of two small containers obtained from step 3 into one container and record the number of small containers that remain.



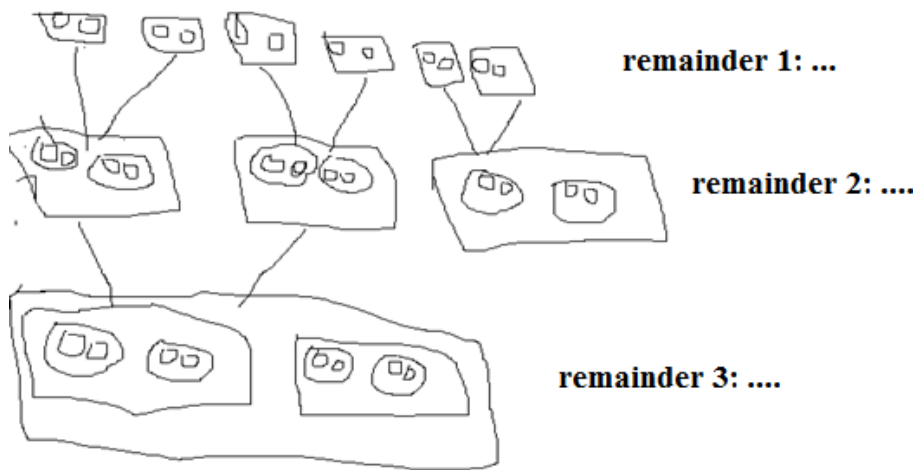
Step 5: Make a group of two containers obtained from step 5 and then, record the number the remained containers.



Step 5: Write the number of groups obtained. Can they be grouped into groups of two? If no, consider this number.

Step 6: Write answers you have gotten from bottom to the top.

g) Data recording



Answer:

Form the number formed by digits from bottom to the top:

Answer, remainder3, remainder2, remainder 1.

What is the number obtained?

h) Interpretation of results/Expected outcome

We expect to have:

- Six groups of two stones in each container (pairs).remainder 1 is 0
- Three groups of containers having two stones each, remainder 2 is 0.
- one group of a container having two containers and keep the remainder as another group: remainder 3 is 1 and the answer is 1.

The obtained number is 1100. Is this number the number 12 changed from base 10 to base 2?

Let's verify: $1100_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 = 12_{10}$ It is correct.

Conclusion

Therefore 12 stones in group of ten (base ten) becomes 1100 in group of two (base two).

It implies $12_{10} = 1100_2$.

PRACTICAL ACTIVITY 2: Factorization of quadratic trinomial

a) Rationale:

This activity is conducted when teaching the factorization of expressions of the type $Ax^2 + Bx + C$. It is taught in unit 5 of S3.

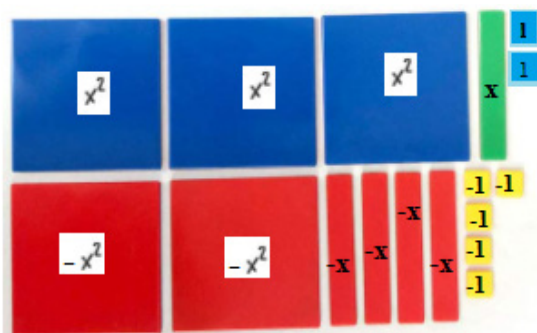
Everyday life and specifically in some fields of study or professions, the quadratic trinomial $Ax^2 + Bx + C$ is used. In finance, a common polynomial equation that comes up is the calculation of present value. This is used in accounting when the present value of assets must be determined. It is used in asset (stock) valuation. It is used in bond trading and mortgage calculations.

b) Objective 1:

Factorize the expression $2x^2 + 7x + 3$

c) Materials required:

Algebraic tiles, plane surface such as a table, pen, or pencils.



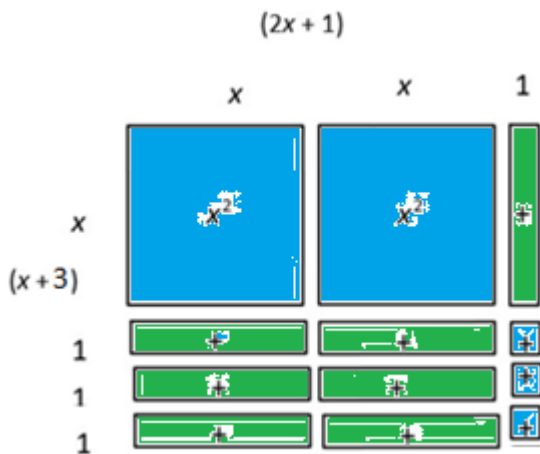
d) Procedures:

Step 1: Represent this expression using $2x^2$ tiles, 7 positive x tiles, and 3 positive integer tiles.



Step 2: Arrange the tiles so that they can make a rectangle or a square: starting with the $2x^2$ tiles.

Step 3: Rearrange the tiles from the step 2, so that tiles on parallel sides are similar one another.



What is the width of the obtained rectangle? What is the length of the obtained rectangle?

Expected answer:

The arranged tiles make a rectangle with width $(x+3)$ and length $(2x+1)$.

Step 4: Find the area of the obtained rectangle by multiplying expressions of width and length to check your answer.

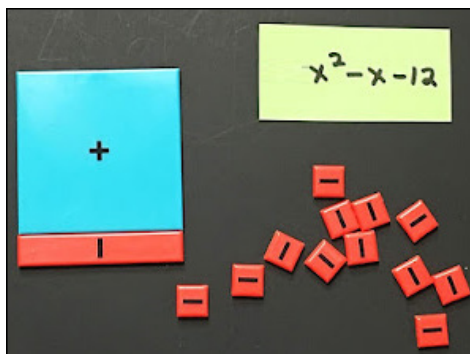
Expected answer:

$$(2x+1)(x+3) = 2x \cdot x + 2x \cdot 3 + 1 \cdot x + 1 \cdot 3 = 2x^2 + 7x + 3.$$

That is why the factors of $2x^2 + 7x + 3$ are $(2x+1)(x+3)$.

Objective 2: Factor the expression $x^2 - x - 12$

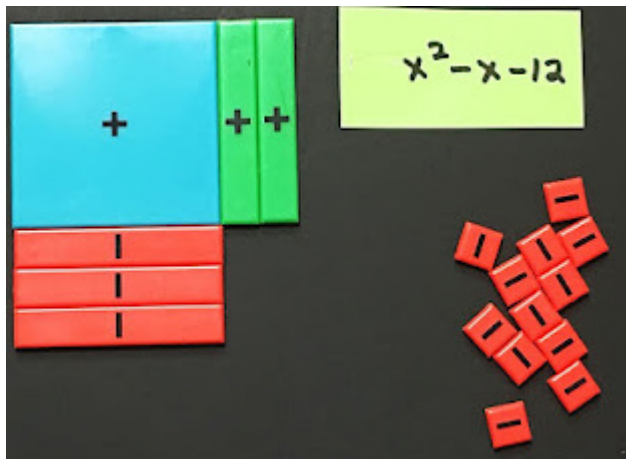
b) Required materials: Algebraic tiles.



c) Procedures:

Step 1: Represent this expression using 1 x^2 tile, 1 negative x tiles, and 12 negative integer tiles.

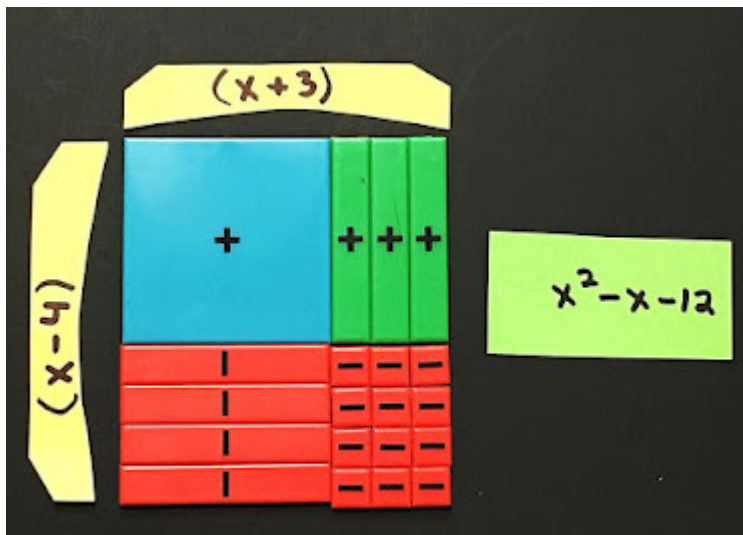
Step 2: Since there was no way to make a rectangle that fit those 12 - tiles, we needed to add in some additional zero pairs (1 of each rectangular + and - x tile). Use 3 rectangular - x tiles and 2 rectangular x tiles.



Is it enough to fit those 12 - tiles? Is the obtained figure a rectangle? How many more rectangular -x tiles are needed?

Step 4: Add 4 rectangular -x tiles and 3 rectangular x tiles to make all 12 - tiles fit.

Does $-4x + 3x = -x$? Verify if the obtained figure is a rectangle. And write its area.



Step 5: Multiply the sides of the rectangle to verify if its area is $x^2 - x - 12$. Then, deduce the *factors* of $x^2 - x - 12$

Expected answer:

$$(x - 4)(x + 3) = x^2 - x - 12$$

e) Interpretation of results and conclusion

i) Basing on your findings, summarize how to use algebraic tiles to factor the quadratic expression

$$Ax^2 + Bx + C.$$

ii) How do you factor a quadratic trinomial involving some negative coefficients?

Expected answer:

i) To factorize the expression of the form $Ax^2 + Bx + C$, the following steps:

- Use algebra tiles to represent a trinomial of the form , where a, b, and c are integers. Include a of the x^2 tiles, b of the x tiles, and c of the integer tiles. If b or c are negative, represent them using the negative (-) tiles.
- Arrange the tiles into a rectangle.
- The factors of the trinomial are represented by the lengths of the sides of the rectangle. Remember to consider whether the x tiles are positive or negative.
- Check your answer. Multiply the factors to make sure they produce the original trinomial.

iii) When we factor quadratic trinomials involving negatives, we may not start and end with the same number of tiles. We may have to add in some zero pairs as we go. In the case of $x^2 - x - 12$, the coefficient of x is -1.

- If there is no way to make a rectangle that fits the given tiles, you can need to add in some additional zero pairs (1 of each rectangular + and - x tile). You can do this because:

$$-1 + 0 = -1 \text{ or}$$

$$-2 + 1 = -1 \text{ or}$$

$$-3 + 2 = -1 \dots \text{ and so on...}$$

- By adding in equal amounts of rectangular + and - x tiles, we are not changing the trinomial's B value. It's still -1.

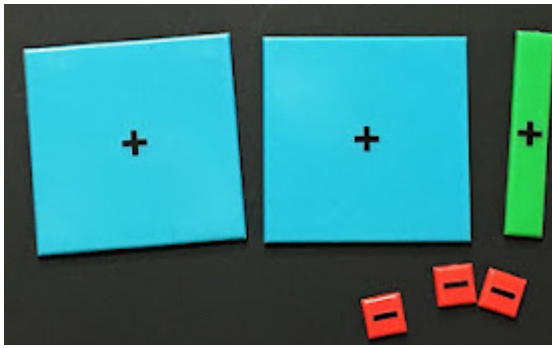
f) Guidance on evaluation

Ask students to use tiles when factoring other trinomials of second order. For example: **Factor $2x^2 + x - 3$**

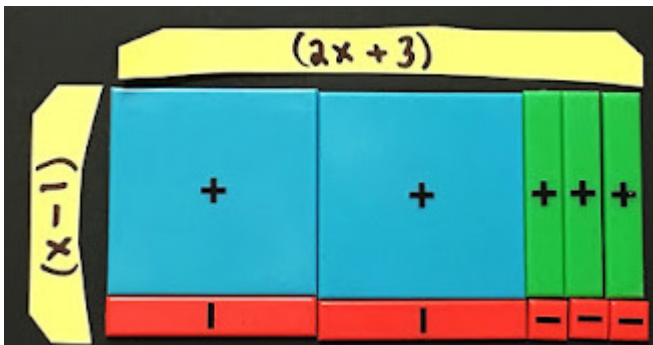
Expected answer:

Using Algebra tiles is so helpful when factoring quadratic trinomials where the A value is greater than 1 and B and/or C are negative. The tiles make the process so much more intuitive!

We needed to start with: 2 x^2 tiles, 1 rectangular x tile and 3 - tiles.



As we see, we do not have enough pieces to make a nice, even rectangle. So we will need to start adding zero pairs to keep B at +1. Here I added a rectangular $+x$ and a rectangular $-x$. This wasn't quite enough to fill it in.



I added one more zero pair of x tiles and our rectangle is complete. Therefore, $2x^2+x-3$ factors to $(x-1)(2x+3)$.

PRACTICAL ACTIVITY/ EXPERIMENT 3: Pythagoras theorem

a) Rationale:

This experiment is done when teaching the Pythagoras theorem. It is taught in unit 8 of S3.

Pythagoras theorem is applied to a right-angled triangle. If the lengths of any two sides of a right-angled triangle are given, then the length of the third side can be obtained.

In real life, the Pythagorean theorem is helpful in security cameras for face recognition, architects use the technique of the Pythagoras theorem for engineering and construction fields, Pythagoras theorem is applied in surveying the mountains, It is also used in navigation to find the shortest route, Painters use ladders to paint on high buildings with the help of the Pythagoras theorem.

b) Objective:

To verify practically that in a right-angled triangle, the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides.

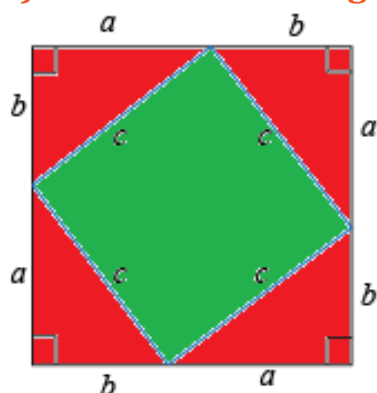
This theorem helps students to be able to understand for example how to calculate the length c of a ladder to be put on a house of length b when it stands at the distance a from the house.

c) Required materials

To conduct this activity, use the following materials:

- Cut-outs of 4 right-angled triangles with sides a , b , and c of the same sizes for which the side a and b form the right angle and the side c is the hypotenuse.
- A square of side c that is equal to the length of the hypotenuse of the triangle.
- Table or a plane surface such as the soil.

d) Illustration or diagram



e) Procedures

Step 1. Place the 4 right angled triangles of the same size on the table or any other plane surface in the order in a way that their hypotenuses form a square of sides c as shown in the illustration above.

Step 2. Place the square of side C in the interior part created by 4 triangles. Does the square fit in that part?

Step 3. Remove the internal square.

Step 4. Discuss how you can find the area of the square of side $a+b$ and the area of the part covered by 4 triangles. If the interior square is removed, how can we determine its area basing on the area of the large square and the area of 4 triangles?

Expected answers:

Now, we are sure that the area of the large square with side $a+b$ is the sum of the area of the interior square and the area for 4 triangles.

The area of the large square: is $(a + b)^2 = a^2 + 2 ab + b^2$

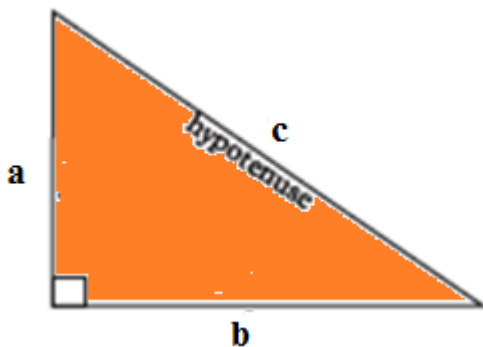
The area of the 4 triangles is: $4 \cdot (ab/2)$

The area of the interior part is the area of the square of side c . It is also the difference between the area of a large square and the area for 4 triangles.

$$c^2 = a^2 + b^2 + 2ab - 4 \cdot (ab/2) = a^2 + b^2$$

f) Interpretation of results and Conclusion

Given the right-angled triangle with side a , b , and c such that c is its hypotenuse, what is the relationship between a , b and c ?



Complete: $c^2 = \dots + \dots$

What will be the length of hypotenuse if the other sides are 3cm and 4cm?

Expected answer

In a right-angled triangle with sides a , b and c where c is hypotenuse, the relationship among the sides is:

$$c^2 = a^2 + b^2$$

This is expressed as Pythagoras theorem: the square of hypotenuse is equal to the sum of the squares of the other two sides.

If $a=3\text{cm}$, $b=4\text{cm}$, we have: $c^2 = (3\text{cm})^2 + (4\text{cm})^2 = 25\text{cm}^2$

Therefore, c equals the square root of 25cm^2 which gives $c = 5\text{cm}$.

If the other sides are 3cm and 4cm , the length of the hypotenuse is 5cm .

g) Guidance on the evaluation

Ask students to use 4 right angled triangles of the same size made from cut-outs of sheets of paper such that the sides that form the right angle are $a = 5\text{cm}$, $b = 12\text{cm}$. invite them to determine the length of the hypotenuse of those triangles.

PRACTICAL ACTIVITY/ EXPERIMENT 4: Median and the hypotenuse in a right triangle

a) Rationale:

This experiment is conducted when teaching the median theorem in a right-angled triangle from unit 8 of S3.

A line segment that joins any vertex of the triangle and the mid-point of its opposite side is called a median. It is also the line from the midpoint of a side to the opposite interior angle. House designers and carpenters use the median theorem when cutting the timbers or iron bars to make triangular objects for houses.

b) Objective:

To verify that the median from the right-angled vertex to the hypotenuse is half the length of the hypotenuse. This activity will help students to understand that the median subdivides the right-angled triangle into two similar isosceles triangles.

c) Materials required:

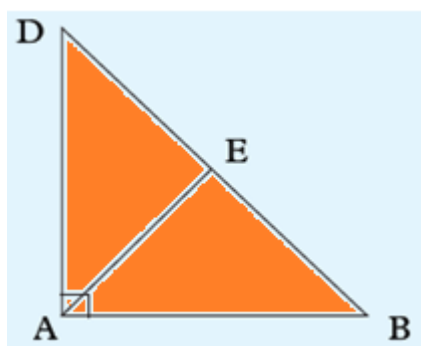
Manila paper, two pair of scissors, protractor, ruler, T-square, timber or triplex, pins, and scotch.

d) Procedures:

Step 1: Take a manilla paper and ensure it has right angles.

Step 2: Fix the manila paper on a timber or triplex.

Step 3: Cut the manila paper into a right angled-triangle ABD rectangle in the angle A as in figure below:



Step 4: Fix the triangular part on the triplex using the scotch and fix the pins on the 3 vertices of the triangle ABD.

Step 5: Use a ruler to locate the point E by which passes the median from the right-angle A hypotenuse.

Step 6: Fix a pin on the point E and use the thread to join them.

Step 7: Measure and compare the length of AE to the hypotenuse DB, and distance DE and EB and record answer in the following table.

| AE | DB | $\frac{DB}{AE}$ | $\frac{EB}{AE}$ | $\frac{DE}{AE}$ |
|----|----|-----------------|-----------------|-----------------|
| | | | | |

What do you notice?

e) Results, interpretation, and conclusion

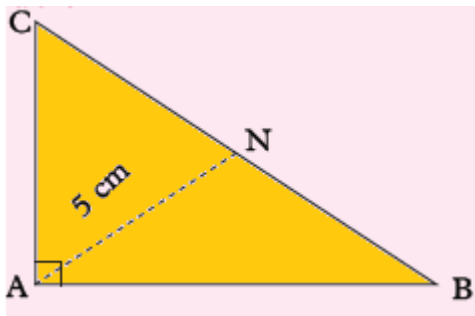
Basing on your answers, in the table above, explain the relationship between the length of the median and the half of the hypotenuse.

Expected answer:

The length of the median from a right angle to the hypotenuse is equal to the half of the hypotenuse. This means that the triangle AEB and AED are isosceles triangles because they have two sides with the same length: sides $AE = DE$ and $AE = EB$.

f) Guidance on the evaluation:

Provide an activity to be done by students. For example: In a right-angled triangular object ABC, side BC is the hypotenuse and AN is 5 cm long.



What is the length of the hypotenuse?

PRACTICAL ACTIVITY/ EXPERIMENT 5: Median of a triangle and the surface area

a) Rationale:

This experiment is conducted when teaching the theorem related to the median and the surface area of a triangle from unit 8 of S3.

A line segment that joins any vertex of the triangle and the mid-point of its opposite side is called a median. It is also the line from the midpoint of a side to the opposite interior angle. House designers and carpenters use the median theorem when cutting the timbers or iron bars to make triangular objects for houses.

b) Objective:

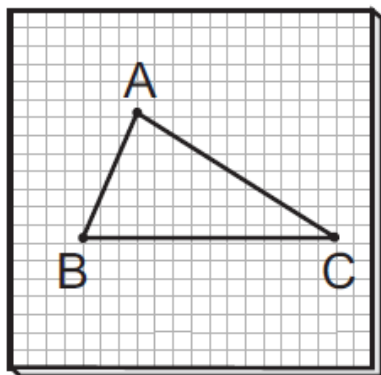
To verify that median of a triangle divides it in two triangles of equal area.

c) Required materials:

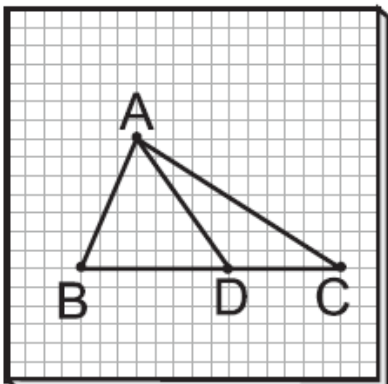
Geoboard, rubber-bands, Geoboard pins.

d) Procedures and diagram

Step 1: Consider three pins on a geoboard and use a rubber band to form the $\triangle ABC$ such that the length of base BC has even number of units.



Step 2: From the mid-point of BC say D , join A using a rubber band to form the median AD



Step 3: Find out the area of triangle ADC and triangle ADB by counting the number of unit squares enclosed in it as discussed earlier.

Step 4: Repeat the above activity by forming different triangles by changing the geoboard pins and find out area of triangles.

Record the findings in the following table:

| No | Area of the triangle ABD | Area of the triangle ADC | Is the area of the triangle ABD equals to the area of ADC? (Yes or No) |
|----|--------------------------|--------------------------|--|
| 1 | | | |
| 2 | | | |
| 3 | | | |

e) Results, interpretation, and conclusion

Basing on your answers, in the table above, explain the relationship between the areas of two triangles made by the median.

Expected answer:

The median of a triangle divides it in two triangles of equal area.

Guidance on the evaluation:

Give students a related problem to be solved.

PRACTICAL ACTIVITY 6: Meaning of different trigonometric ratios

a) Rationale:

This activity is conducted when teaching the trigonometric ratios from unit 8 of S3.

Trigonometry can be used to roof of a house, to make the roof inclined (in the case of single individual bungalows) and the height of the roof in buildings etc. It is used in naval and aviation industries, and in cartography (creation of maps).

b) Objective:

Explore the meaning of different trigonometric ratios.

c) Required materials:

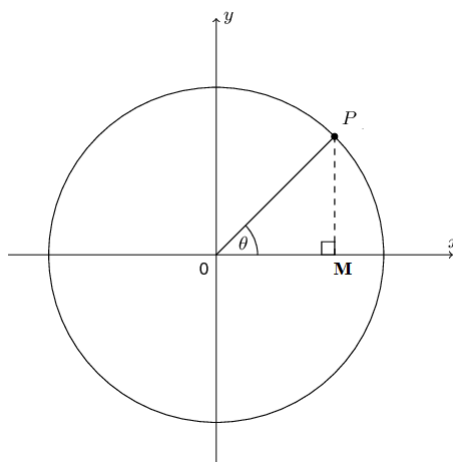
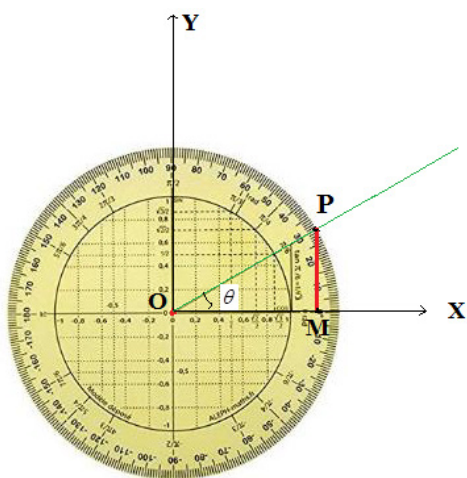
Manila paper, pair of compasses, Transparent circular protractor, wire needle, T-square, ruler

d) Procedures:

Step 1: Choose a point O and mark it on the manila paper.

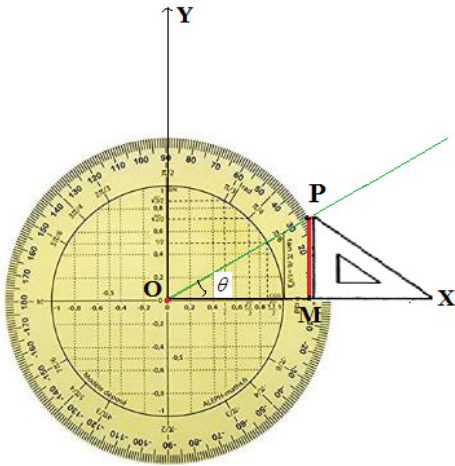
Step 2: Draw axes of a cartesian plane that intersect in the point O.

Step 3: Choose an angle θ and measure its value using a protractor and mark the point P such that $\theta = \angle POX$ where OX is the horizontal line.



Step 4: Place the T-square such that one edge of the set square (other than the hypotenuse) coincides with the horizontal line.

Step 5: Now slide the set square along the horizontal line so that other edge of the set square forming the right angle passes through the point P as shown in the following figure:



Step 6: Mark the point M on the vertex of the T-square to represent a perpendicular PM on horizontal line OX.

Step7: Now Use a ruler to measure length of PM and OM. You can project the segment PM to Y-axis to get its exact value read on the circular protractor.

Step 8: Define sine, cosine and tangent of the angle θ if :

$$\sin \theta = \frac{PM}{OP}, \quad \cos \theta = \frac{OM}{OP} \quad \text{and} \quad \tan \theta = \frac{PM}{OM}$$

Expected answer:

$$\sin \theta = \frac{PM}{OP} = \frac{\text{Opposite side}}{\text{Hypotenuse}} \quad \cos \theta = \frac{OM}{OP} = \frac{\text{adjacent side}}{\text{Hypotenuse}}, \quad \tan \theta = \frac{PM}{OM} = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\text{Or } \tan \theta = \frac{PM}{OM} = \frac{\sin \theta}{\cos \theta}$$

Step 8: Repeat the activity by making different angles by varying the position of the needle and complete the following table:

| No | Value of the angle θ | PM | OM | $\sin \theta$ | $\cos \theta$ | $\tan \theta$ |
|----|-----------------------------|----|----|---------------|---------------|---------------|
| | | | | | | |

e) Results, interpretation, and conclusion

What are the main trigonometric ratios did you explore?

What does each trigonometric ratio mean?

Expected answers:

Given an angle θ , the trigonometric ratios sine, cosine and tangent of the angle θ are defined as follows:

$$\sin \theta = \frac{PM}{OP} = \frac{\text{Opposite side}}{\text{Hypotenuse}} \quad \cos \theta = \frac{OM}{OP} = \frac{\text{adjacent side}}{\text{Hypotenuse}}, \quad \tan \theta = \frac{PM}{OM} = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\text{Or } \tan \theta = \frac{PM}{OM} = \frac{\sin \theta}{\cos \theta}$$

f) Guidance on the evaluation

Ask students to consider the angle $\theta = 45^\circ$ and find cosine and tangent of the angle $\theta = 45^\circ$

PRACTICAL ACTIVITY 7: Trigonometric ratio of an angle and the variation of lengths of the sides of the triangle

a) Rationale:

This activity is conducted when teaching the trigonometric ratios for an angle of a triangle whose sides vary in lengths. It is taught in unit 8 of S3.

Trigonometry can be used when preparing triangular parts of roof of a house, to make the roof inclined (in the case of single individual bungalows) and the height of the roof in buildings etc. It is used in naval and aviation industries.

b) Objective:

To verify that the values of trigonometric ratios of an angle do not vary with the lengths of the sides of the triangle.

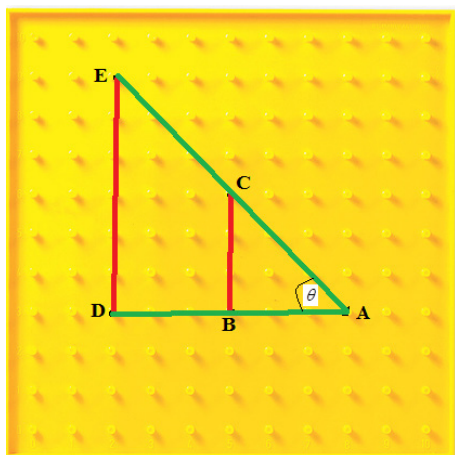
c) Materials required:

Geoboard, rubber bands, ruler, pencils, and pens.

d) Procedures and diagram:

Step 1: Consider 5 geoboard pins on the geoboard at suitable points A, B, C, D and E and join them with 2 rubber bands of different colours to represent two similar right triangles

as shown in the following figure:



Step 2: Measure length of AB, BC, AD, DE, AC and AE using a ruler.

Step 3: Find:

$$\frac{BC}{AC}, \frac{AB}{AC}, \frac{BC}{AB}, \frac{DE}{AE}, \frac{AD}{AE}, \frac{DE}{AD}$$

Step 4: Repeat the activity by making other pairs of similar right triangles and complete the following table:

| No | BC | AC | DE | AE | $\frac{DE}{AE}$ | $\frac{BC}{AB}$ | AB | AD | $\frac{DE}{AE}$ | $\frac{BC}{AB}$ |
|------------|----|----|----|----|-----------------|-----------------|----|----|-----------------|-----------------|
| θ_1 | | | | | | | | | | |
| θ_2 | | | | | | | | | | |
| θ_3 | | | | | | | | | | |

e) Results, interpretation, and conclusion

Compare the length of BC and DE, what are different ways in which you can determine the sine of the angle θ ? What about cosine of the angle θ ?

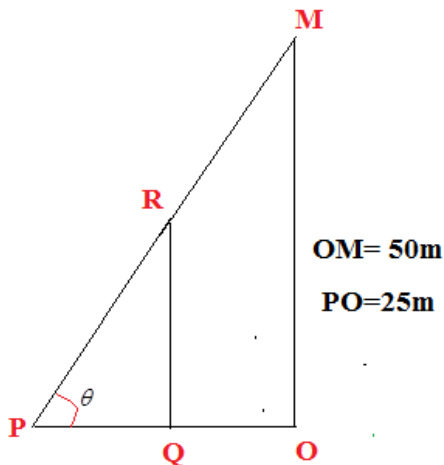
Expected answers:

We see that $\sin \theta = \frac{DE}{AE} = \frac{BC}{AC}$. However, DE is greater than BC and AE is greater than AC.

Therefore, the values of trigonometric ratios of an angle do not vary with the lengths of the sides of the triangle.

f) Guidance on the evaluation:

Ask students to use a right-angled triangle whose sides were increased. Invite them to use a ruler to determine the tangent of its angle θ .



PRACTICAL ACTIVITY 8: Standard trigonometric identity

a) Rationale:

This practical activity is conducted when teaching the trigonometric identities. It is taught in unit 8 of S3.

Trigonometry simply means calculations with triangles (that's where the tri comes from). It is a study of relationships in mathematics involving lengths, heights, and angles of different triangles. Trigonometry spreads its applications into various fields such as architects, surveyors, astronauts, physicists, and engineers. In architecture, when developing large infrastructure. The six different identities are used to find either the length of one or more sides of a shape, or the angle at which different materials should be placed at. It is common to find them when constructing blueprints for actual structures. The reason that trigonometric identities are so important to architecture is that it helps you be as accurate as possible when determining sizes and proportions. This lets you know how much space you have to build, as well as what is the most efficient way of building something so that you maximize space, while minimizing cost.

b) Objective:

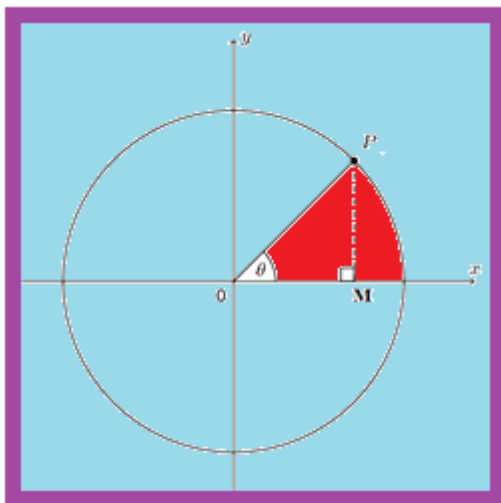
To verify standard trigonometric identity $\cos^2 \theta + \sin^2 \theta = 1$

c) Required materials:

Circular board, wire needle, set square, T-strips, ruler.

d) Procedures:

Step 1: Fix the wire needle at the centre O of the circular board of radius one unit to make an angle $POX = \theta$ with the horizontal line as shown in figure below:



Step 2: Observe the triangle APM such that PM is perpendicular to OX in the

point M.

Step 3: Use Pythagoras theorem to determine the square of the hypotenuse OP.

Complete $\cos \theta = \dots$ and $\sin \theta = \dots$

e) Results, interpretation, and conclusion

Replace the side OM and PM by trigonometric expression in the equality you have found.

What is the identity do you get?

Expected answer

By the Pythagoras theorem, we get $OP^2 = OM^2 + PM^2$

Complete $\cos \theta = \frac{OM}{OP} = \frac{OM}{1} = OM$ and $\sin \theta = \frac{PM}{OP} = \frac{PM}{1} = PM$

Therefore, $OP^2 = OM^2 + PM^2$ means that $1 = 1 = \cos^2 \theta + \sin^2 \theta$

f) Guidance on the evaluation:

Ask students to proceed in the same way to verify that $1 + \tan^2 \theta = \frac{1}{\cos^2 \theta}$.

PRACTICAL ACTIVITY/ EXPERIMENT 9: Area of a circle

a) Rationale:

This experiment is conducted when teaching elements of a circle and disk. It is taught in unit 9 of S3.

In real life, many shapes (shapes of objects) around us have the circular form. An object with a circular face such as a dining plate, a pop can, a traffic cone, or a circular flower bed in a garden are all built or prepared keeping the concept of area of a circle in mind.

b) Objective:

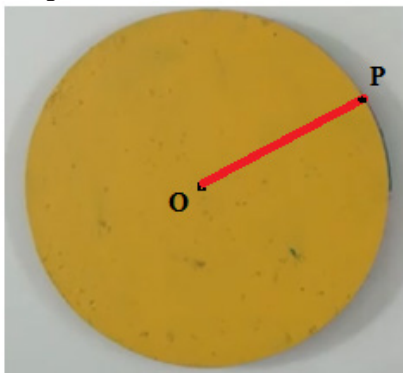
Explore the area πr^2 of a circle of radius r using sectors of the circle

c) Required materials:

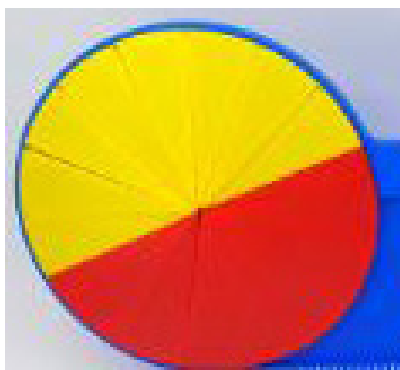
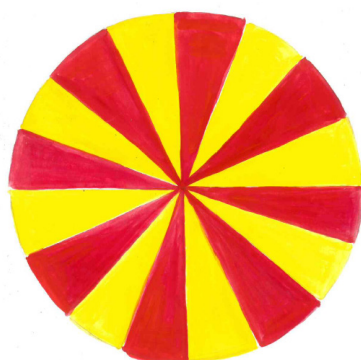
Pens, Circle – Area and Diameter Demonstrator: 1 plastic demonstrator board of 48 cm × 25 cm. It consists of 17 sectors: 15 sectors that are equal to $1/16$ of the cylinder volume; and 2 sectors that are equal to $1/32$ of the cylinder volume.

d) Procedures:

Step 1: Take a circular disk of radius r and discuss how to find its circumference.

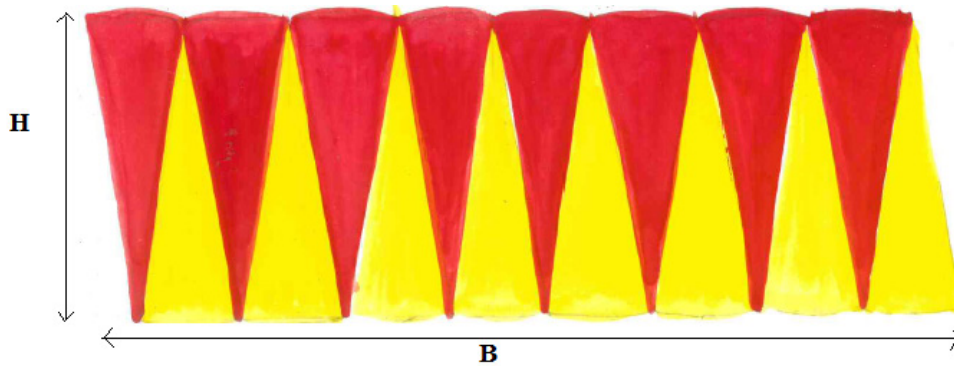


Step 2: Take a similar circular cut-out which is divided into 16 cut-outs of equal sectors, half of which 8 are in red colour others are in yellow colour as shown in the following figure:



What is the figure formed by this the exterior part? What is its radius? How can you find its circumference?

Step 3: Arrange these sectors to form a shape as shown below where arc of sectors with the same colour must be together.



What is the new figure formed by sectors? Compare the length of its base with the circumference of the original circle. What is its height?

How can we find its area? compose its area with the area of the original circle

e) Results, interpretation, and conclusion

How can you form the area of the new figure? Is it the same area of the original circle? Explain the original way of finding the area of the respective circular cut-out.

Expected answers:

The figures which is so formed above looks like a parallelogram having length equal to half of circumference of respective circular cut-out and height equal to radius of circular cut-out.

Hence, we get

$$\begin{aligned}\text{Area of circle} &= \text{Area of Parallelogram} \\ &= \text{Length of parallel sides} \times \text{Height of parallelogram} \\ &= \pi r^2\end{aligned}$$

f) Guidance on the evaluation

Ask students to repeat the activity by taking 32 equal cut-outs of sectors of the circular sheet of manila paper whose radius is known $r=25\text{cm}$.

PRACTICAL ACTIVITY/EXPERIMENT 10: Angle subtended at the centre of a circle by unequal chords

a) Rationale:

This experiment is conducted when teaching the angles at the centre and the circumference of a circle from unit 9 of S3.

In real life, many shapes (shapes of objects) around us have the circular form. An object with a circular face such as a dining plate, a pop can, a traffic cone, or a circular flower bed in a garden are all built or prepared keeping the concept of area of a circle in mind. We need to know the relationship between the angles at the centre and the circumference of such circular objects.

b) Objective:

To verify that a longer chord subtends greater angle at the centre of a circle.

c) Material required:

Circular board or circular part of a manila paper, scotch, small ruler, thread, full protractor.

d) Procedures:

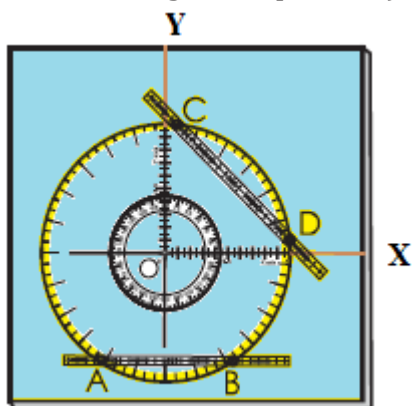
Step 1: Take a pair of compasses and draw a circle of centre O on the manila paper.

Step 2: Draw a cartesian plan with origin O (where axes intersect), and graduate axes

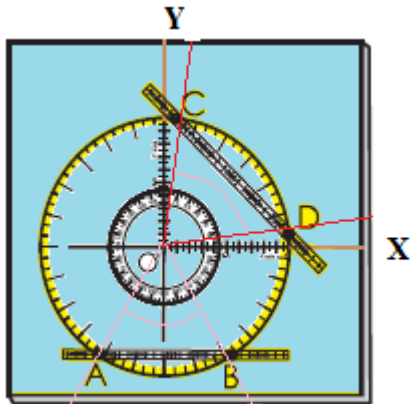
Step 3: Choose two points C and D on the circle such that CD is a chord.

Step 4: Take one plastic strip or small ruler and place it on the circular part of the manila paper representing the chord CD of the circle.

Step 5: Take another plastic strip or small ruler and place it on the circular part of the manila paper representing another chord AB (different from CD in length and position) of the circle.



Step 6: With the help of Pencils of different colours draw two angles at the centre O subtended by the two chords as shown in the following figure:



Step 7: Measure the angle AOB and the angle COD at the centre of circle with the help of markings on the circular part of the manila paper or a full protractor.

Step 4: Repeat the above activity by taking different lengths of the chords and complete the following table:

| No | AB | CD | Angle AOB | Angle COD | Larger chord: Yes or No | Larger angle: Yes or No |
|----|----|----|-----------|-----------|----------------------------|----------------------------|
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | | | | |

e) Results, interpretation, and conclusion

Basing on your findings, complete by small or large

When the chord is small, it subtends a angle at the center.

When the chord is long, it subtends a angle at the center.

Expected answer:

On a same circle, a longer chord subtends greater angle at the centre of this circle.

f) Guidance on the evaluation:

Ask students to draw a circle of centre O and radius 40cm. Invite them to take two different chords one of 20cm and the other of 30cm. let them measure and compare the values of angles subtended by the chords at the centre of circle.

PRACTICAL CTIVITY/ EXPERIMENT 11: Angles subtended at the centre of a circle by equal chords

a) Rationale:

This experiment is conducted when teaching the relationship between angles subtended at the centre of the circle by equal chords taken of the same circle. It is taught in unit 9 of S3.

In real life, many shapes (shapes of objects) around us have the circular form. An object with a circular face such as a dinning plate, a pop can, a traffic cone, or a circular flower bed in a garden are all built or prepared keeping the concept of area of a circle in mind. We need to know the relationship between the angles subtended at the centre of the circle by equal chords of such circular objects.

a) Objective:

To verify that equal chords subtend equal angles at the centre of the circle.

b) Material required:

Circular board or circular part of a manila paper, scotch, small ruler, thread, pins, full protractor.

c) Procedures:

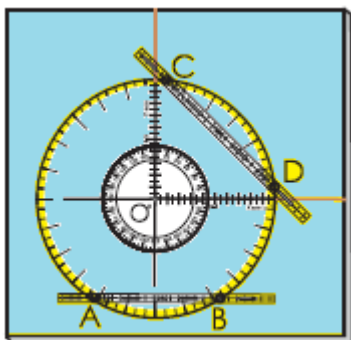
Step 1: Take a pair of compasses and draw a circle of centre O on the manila paper.

Step 2: Draw a cartesian plane with origin O (where axes intersect) and graduate axes.

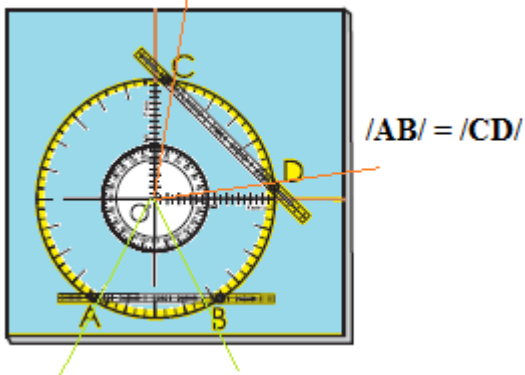
Step 3: Choose two points C and D on the circle such that CD is a chord.

Step 4: Take one plastic strip or small ruler and place it on the circular part of the manila paper representing the chord CD of the circle.

Step 5: Take another plastic strip or small ruler and place it on the circular part of the manila paper representing another chord AB of the same length as CD but in different positions of the circle.



Step 6: With the help of Pencils of different colours draw two angles at the centre O subtended by the two chords as shown in the following figure:



Step 7: Measure the angle AOB and the angle COD at the centre of circle with the help of markings on the circular part of the manila paper or a full protractor.

Step 4: Repeat the above activity by taking different lengths of the chords and complete the following table:

| No | AB | CD | Angle AOB | Angle COD | Is angle AOB=Angle COD? |
|----|----|----|-----------|-----------|-------------------------|
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |

d) Results, interpretation, and conclusion

Basing on your findings, complete

If the chord AB has the same length as the chord CD taken on the same circle of center O, the angle AOB and the angle COD subtended at the centre of the circle are.....

Expected answer:

If the chord AB has the same length as the chord CD taken on the same circle of center O, the angle AOB and the angle COD subtended at the centre of the circle are.....

Therefore, equal chords subtend equal angles at the centre of the same circle.

PRACTICAL ACTIVITY/ EXPERIMENT 12: Angle subtended at the centre and at any point on the circumference of circle

a) Rationale:

This experiment is conducted when teaching the angles subtended at the centre and at the circumference of a circle. It is taught in unit 9 of S3.

In real life, many shapes (shapes of objects) around us have the circular form. An object with a circular face such as a dining plate, a pop can, a traffic cone, or a circular flower bed in a garden are all built or prepared keeping the concept of area of a circle in mind. We need to know the relationship between the angles subtended at the centre and at the circumference of such circular objects.

b) Objective:

To verify that an angle subtended by an arc of a circle at the centre, is double the angle subtended by it on any point on the circumference of circle.

c) Materials required:

Circular board or circular part of a manila paper, scotch, small ruler, thread, pins, full protractor.

d) Procedures:

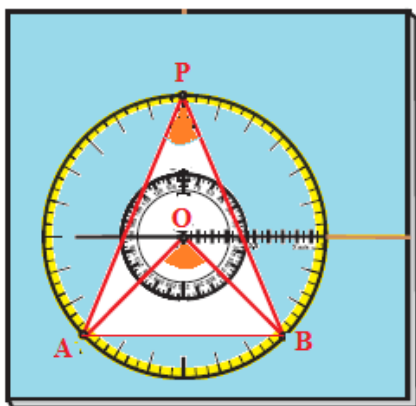
Step 1: Take a pair of compasses and draw a circle of centre O on the manila paper.

Step 2: Choose two points A and B on the circle such that AB is a chord.

Step 3: Use a plastic strip or small ruler and place it on the circular part of the manila paper representing the chord AB of the circle.

Step 5: Take another point P of the circle.

Step 6: Use a thread or a pencil to form the angle AOB and the angle APB.



Step 6: Measure the value of the angle AOB and the value of the angle APB and compare them.

Step 7: Repeat the activity by taking point P at different positions on the boundary of circle on circular board.

Step 8: Repeat the activity by varying length of arc AB and Complete the following table:

| No | Angle AOB | Angle APB | Angle AOB \div (Angle APB) ? |
|----|-----------|-----------|--------------------------------|
| 1 | | | |
| 2 | | | |
| 3 | | | |

e) Results, interpretation, and conclusion

Basing on your findings, complete

Angle AOB = ... APB

Expected answer:

$AOB = 2 APB$

If AB is a chord of a circle of centre O, and P another point of this circle, the angle AOB subtended by an arc of a circle at the centre, is double the angle APB subtended by it on any point P of the circumference of circle.

f) Guidance on the evaluation

Ask students to consider the case where the point P lies outside the circle, and verify if $AOB = 2 APB$

They can also consider the case where the point P lies within the circle, and verify if

$AOB = 2 APB$

PRACTICAL ACTIVITY/ EXPERIMENT 13: Angles in the same segment of a circle

a) Rationale:

This experiment is conducted when teaching angles subtended by the same chord at any point on the circumference of a circle. It is taught in unit 9 of S3.

In real life, many shapes (shapes of objects) around us have the circular form. An object with a circular face such as a dining plate, a pop can, a traffic cone, or a circular flower bed in a garden are all built or prepared keeping the concept of area of a circle in mind. We need to know the relationship between angles subtended by the same chord at any point on the circumference of such circular objects.

b) Objective:

To verify that the angles in the same segment of a circle are equal.

c) Material required:

Circular board or circular part of a manila paper, scotch, small ruler, thread, pins, full protractor.

d) Procedures:

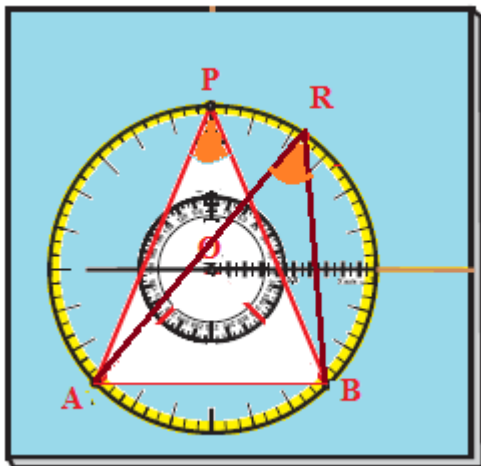
Step 1: Take a pair of compasses and draw a circle of centre O on the manila paper.

Step 2: Choose two points A and B on the circle such that AB is a chord.

Step 3: One plastic strip or small ruler and place it on the circular part of the manila paper representing the chord AB of the circle.

Step 5: Take other two different points P and R of the circle.

Step 6: Use a thread or a pencil to form the angle APB and the angle ARB.



Step 6: Measure the value of the angle APB and angle ARB and compare them.

Step 7: Repeat the activity by taking point P and R at different positions on the boundary of circle on circular board.

Step 8: Repeat the activity by varying length of arc AB and Complete the following table:

| No | Angle APB | Angle ARB | Is angle APB = Angle ARB? (Yes or NO) |
|----|-----------|-----------|---------------------------------------|
| 1 | | | |
| 2 | | | |
| 3 | | | |

e) Results, interpretation, and conclusion

Basing on your findings, complete by $<$, $>$ or $=$ to compare the values of two angles

APB ... ARB

Expected answer:

The value of angle APB equals the value of angle ARB.

Therefore, If AB is a chord of a circle of centre O, and P and R other points of this circle, the angle APB and ARB subtended by the same chord AB of the circle have the same values.

f) Guidance on the evaluation

Ask students to consider the case where the point P lies on the circle while the point R lies outside the circle and verify if $\angle APB = \angle ARB$.

PRACTICAL ACTIVITY/ EXPERIMENT 14: The angle in a semi-circle

a) Rationale:

This experiment is conducted when teaching the angles in a semi-circle. It is a lesson from unit 9 that helps students to be able to find the centre of a circle when they have a T-square.

In real life, many shapes (shapes of objects) around us have the circular form. An object with a circular face such as a dining plate, a pop can, a traffic cone, or a circular flower bed in a garden are all built or prepared keeping the concept of area of a circle in mind. We need to know the angles in the half of such circular objects.

b) Objective:

To verify that an angle in a semi-circle is a right angle.

c) Material required:

Circular board or circular part of a manila paper, scotch, small ruler, thread, pins, full protractor, Set square.

d) Procedures:

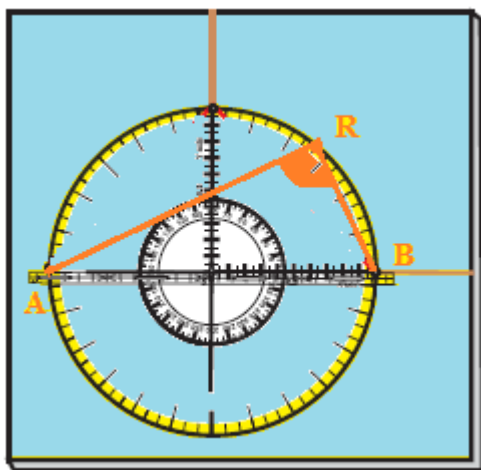
Step 1: Take a pair of compasses and draw a circle of centre **O** on the manila paper.

Step 2: Choose two points **A** and **B** on the circle such that **AB** is a diameter. (chord that passes through the center of the circle)

Step 3: one plastic strip or small ruler and place it on the diameter **AB** of the circular part of the manila paper.

Step 4: Take another points **R** of the circle.

Step 6: Use a thread or a pencil to form the angle **ARB**.



Step 6: Measure the value of the angle **ARB** using a protractor.

Step 7: Repeat the activity by taking a point **R** at different positions on the boundary of circle on circular board.

Step 8: Repeat the activity by varying length of arc **AB** and Complete the following table:

| No | AB | Angle ARB |
|----|----|-----------|
| 1 | | |
| 2 | | |
| 3 | | |

e) Results, interpretation, and conclusion

Basing on your findings, complete:

If A and B and R are points of a circle such that AB is a diameter, the value of the angle ARB isdegrees.

Expected answer:

The value of angle ARB is 90 degrees.

Therefore, If A, R and B are points of a circle such that AB passes through the centre of the circle, the angle ARB in a semi-circle is a **right angle**.

f) Guidance on the evaluation

Ask students to consider the case where AB is not a diameter and then verify if the angle ARB is a right angle.

PRACTICAL ACTIVITY/ EXPERIMENT 15: Line through the centre of a circle bisecting a chord

a) Rationale:

This experiment is conducted when teaching properties of chords from unit 9. In real life, it can help students to identify a centre of a circular object by using a set square.

b) Objective:

Verify that a line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.

c) Required material:

Circular board or circular part of a manila paper, scotch, small ruler, thread, pins, full protractor, Set square.

d) Procedures:

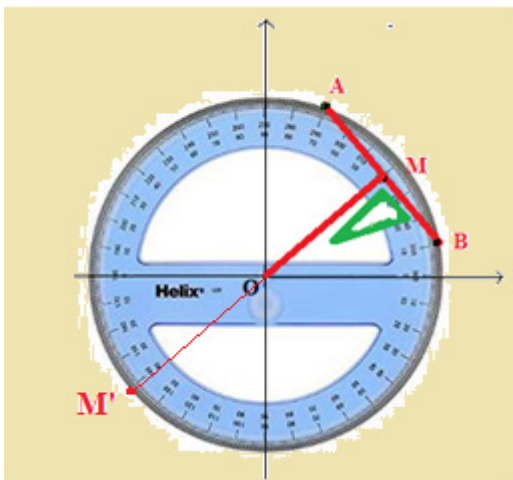
Step 1: Take a pair of compasses and draw a circle of centre O on the manila paper.

Step 2: Choose two points A and B on the circle such that AB is a chord of the circle and measure the length of AB .

Step 3: Mark the point M as the mid-point of the chord AB .

Step 4: Use a ruler or a thread to join the centre O of the circle and the point M .

Step 5: Use a set square or a protractor to measure the value of the angle OMB and the angle OMA .



Step 6: Repeat the above activity by forming chords of different length by fixing the ruler at different positions and complete the following table:

| No | Value of the angle OMA | Value of the angle OMB |
|----|------------------------|------------------------|
| 1 | | |
| 2 | | |
| 3 | | |

e) Results, interpretation, and conclusion

Basing on your findings, complete:

The value of the angle OMB or OMA made by a segment from the centre O of the circle to the mid-point of the chord AB is degrees.

Expected answer:

The value of angle OMB is 90 degrees.

Therefore, if A, and B are points of a circle such that AB is a chord of the circle of centre O, the line drawn through the centre to bisect a chord is perpendicular to the chord AB. This means that if OMA is a right angle, the point M is the mid-point of the chord AB.

f) Guidance on the evaluation

Ask students to consider the case where the point M is not the mid-point of AB and invite them to verify if the angle OMA is a right angle.

PRACTICAL ACTIVITY 16: Centre of circular object

a) Rationale:

This practical activity is conducted when teaching how to locate the centre of a circular object as it is provided in unit 9. In real life, it can help students to identify a centre of a circular object by using a set square.

b) Objective:

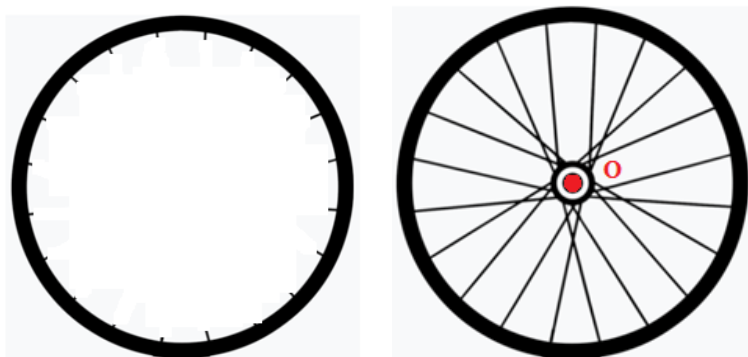
To locate the centre of a circular object in real life.

c) Required material:

Empty bicycle wheel, a piece of chalk, a plane surface such as playground, a meter ruler, a set square.

d) Procedures:

Step 1: Observe the circular object such as a bicycle wheel and discuss in group how to locate the point O centre of a bicycle wheel.



Step 2: Place a circular wheel on a plane surface such as the playground.

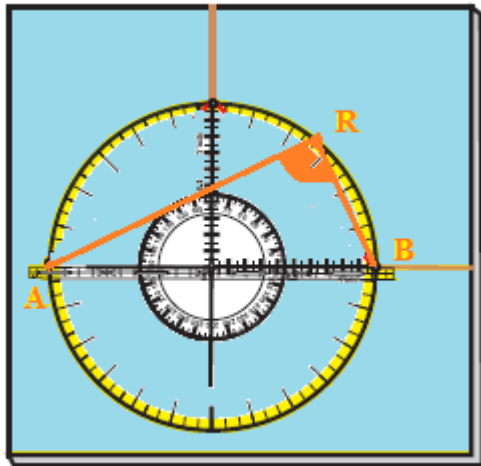
Step 3: Use a piece of chalk to draw a circle surrounding the wheel.



Step 4: remove the wheel and remain with the circle only.

Step 5: Draw a chord RB of the circle and measure the length of RB.

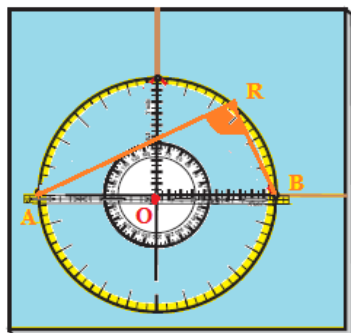
Step 6: Apply the property of a right angle on the point R of the circle to find the point A of the circle such that the segment AB is a diameter of the circle.



Step 7: Use a ruler to measure the length of the diameter AB.

Where is the center of the circle?

Step 8: Fix a nail on the point O which is the mid-point of the diameter AB.



Angle ARB is
90 degrees

$AO = OB$

Step 9: Put the wheel in the initial position and identify its central point.

e) Results, interpretation and conclusion

Basing on the findings, what is the center of the wheel? What is the property did you use to find the central point of the wheel?

Expected answer:

The point O which is the mid-point of the diameter AB is the central point of the wheel. To find the central point we apply the property saying that the angle ARB in a semi-circle, such that AB is a diameter, is a right angle.

PRACTICAL ACTIVITY/ EXPERIMENT 17: Angles in a cyclic quadrilateral

a) Rationale:

This experiment is conducted when teaching properties of angles in a cyclic quadrilateral from unit 9 of S3.

In real life, many shapes (shapes of objects) around us have the circular form. An object with a circular face such as a dining plate, a pop can, a traffic cone, or a circular flower bed in a garden are all built or prepared keeping the concept of area of a circle in mind. House designers need to know the relationship between angles a quadrilateral made from such circular objects.

b) Objective:

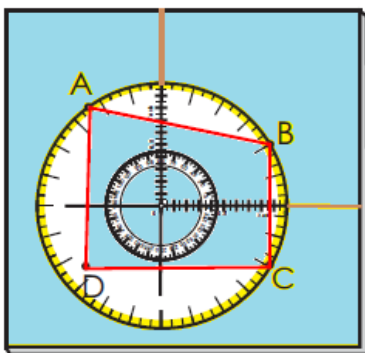
To verify that the opposite interior angles of a cyclic quadrilateral are supplementary or add up to 180° .

c) Materials required:

Circular board or a full protractor fixed on a cardboard, pins, a scotch, rubber bands, two half protractors.

d) Procedures:

Step 1: Fix 4 pins at suitable points A, B, C and D on the boundary of circle on the circular board and form a cyclic quadrilateral ABCD with the help of the rubber band as shown in the following figure:



Step 2: Consider opposite angles in the cyclic quadrilateral ABCD and measure their values with a protractor. Start by the angle ABC and the angle ADC.

Step 3: Find the sum of opposite angles in the cyclic quadrilateral ABCD.

Step 4: Also measure the angle BAD and the angle BCD with the help of half protractor and find their sum.

Step 5: Repeat the activity by forming other cyclic quadrilaterals by changing the position of pins on the circular board and complete the following table:

| No | Angle ABC | Angle ADC | Angle ABC + angle ADC | Angle BCD | Angle BAD | Angle BCD + Angle BAD |
|----|-----------|-----------|-----------------------|-----------|-----------|-----------------------|
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | | | | |

e) Results, interpretation, and conclusion

What is a cyclic quadrilateral?

In the cyclic quadrilateral ABCD, opposite angles are: ... and, the sum of their value is.....

Other opposite angles are: ... and, the sum of their value is.....

Expected answer:

A cyclic quadrilateral is a quadrilateral whose vertices all lie on a circle. The distinctive property of a cyclic quadrilateral is that its opposite angles are supplementary. In the cyclic quadrilateral ABCD, the Sum of each pair of opposite angles 180 degrees.

f) Guidance on the evaluation:

Ask students to verify this property by considering the case in which one of the points A, B, C and D is not on the boundary of circle on circular board and verify whether the sum of opposite angles is 180 degrees.

PRACTICAL ACTIVITY/EXPERIMENT 18: Sum of a pair of opposite angles of a non-cyclic quadrilateral

a) Rationale:

This experiment is conducted when teaching angles in a cyclic quadrilateral. It will help students to be able to establish the relationship between angles of quadrilateral formed basing on a circular object.

b) Objective:

To verify that the sum of each pair of opposite angles of a non-cyclic quadrilateral is not equal to 180° .

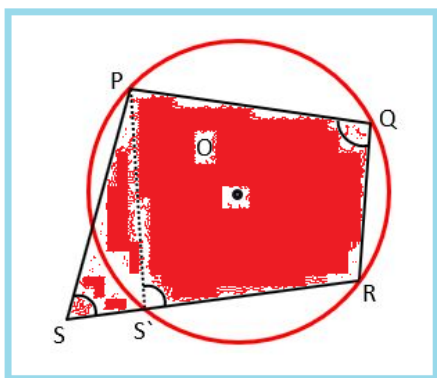
c) Required materials:

Circular board or a full protractor fixed on a cardboard, plastic strips or small rulers, connectors (for circular board), connectors (for strip) or scotch, rubber bands, two half protractors.

d) Procedures:

Step 1: Fix 4 connectors at points P, Q, R and S on the circular board such that 3 connectors are on the boundary of circle and the fourth connector is outside the boundary of circle.

Step 2: Form a quadrilateral PQRS using rubber band as shown in the following figure:



Step 3: Measure the angle PSR and the angle PQR with the help of a half protractor and find their sum.

Step 4: Similarly, measure the angle SPQ and QRS with the help of half protractor and find their sum. Repeat the activity by taking different positions of points and complete the following table:

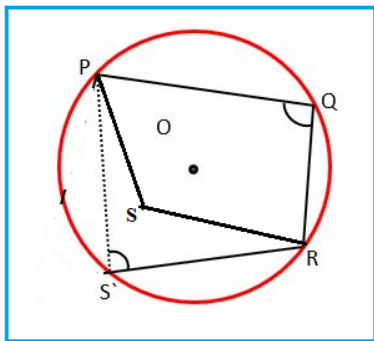
| No | Angle PSR | Angle PQR | Angle PSR + angle PQR | Angle SPQ | Angle QRS | Angle SPQ + Angle QRS |
|----|-----------|-----------|-----------------------|-----------|-----------|-----------------------|
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | | | | |

Is the angle PSR + the angle PQR = 180° ?

Is the angle SPQ + the angle QRS = 180° ?

What happens if you consider the point S' of the circle instead of the point S?

Step 5: Now again fix four connectors on the circular board such that 3 connectors are on the boundary and the fourth connector is **inside the boundary** as shown in the following figure:



Step 6: Measure angle PSR and angle PQR using a half protractor and find their sum.

Step 7: Similarly, measure the angle QPS and the angle QRS by half protractor and find their sum. Repeat the activity by varying positions of points and complete the following table:

| No | Angle PSR | Angle PQR | Angle PSR + angle PQR | Angle SPQ | Angle QRS | Angle SPQ + Angle QRS |
|----|-----------|-----------|-----------------------|-----------|-----------|-----------------------|
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | | | | |

e) Interpretation of results and conclusion:

Is the angle PSR + the angle PQR = 180° ?

Is the angle SPQ + the angle QRS = 180° ?

What happens if you consider the point S' of the circle instead of the point S?

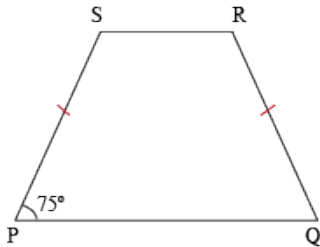
In the 3 cases above explored, mention the case in which PSR + the angle PQR = 180° and the case in which PSR + the angle PQR is different from 180°

Expected answer:

The sum of each pair of opposite angles of cyclic quadrilateral is equal to 180 degrees but when the quadrilateral is not cyclic (when vertexes are not on the boundary of a circle) the sum of opposite angles is not equal to 180°.

f) Guidance on the evaluation

Give students a cyclic quadrilateral PQRS and ask them to determine the measure of the angle PSR as given for example below:



PRACTICAL ACTIVITY 19: Similar polygons

a) Rationale:

This practical activity is conducted when teaching properties of similar polygons. It is taught in unit 9 of S3.

In real life, any form of tiling involves polygons. The tiles need to be similar or to tessellate to cover an area without leaving any gaps. This is directly connected to the angle properties of polygons. Architects include polygons with every plan of a house because rooms usually have 90° corners, but not always. Rooms on a plan are polygons.

The cost of building any structure depends on the lengths of the walls and the size of the angles, these are properties of polygons in real life.

b) Objectives:

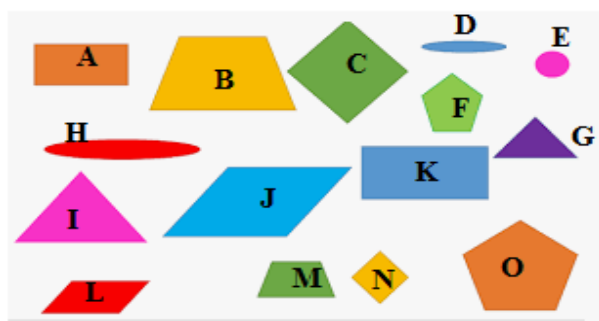
To identify similar shapes in a variety of shapes

c) Required materials:

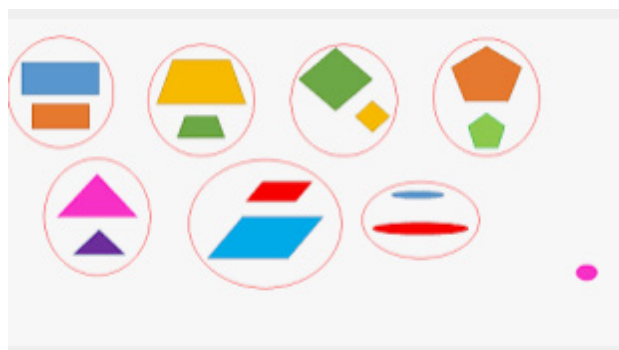
objects such as toys or geometric figures of different types but with similar ones where some are of bigger size than others.

d) Procedures:

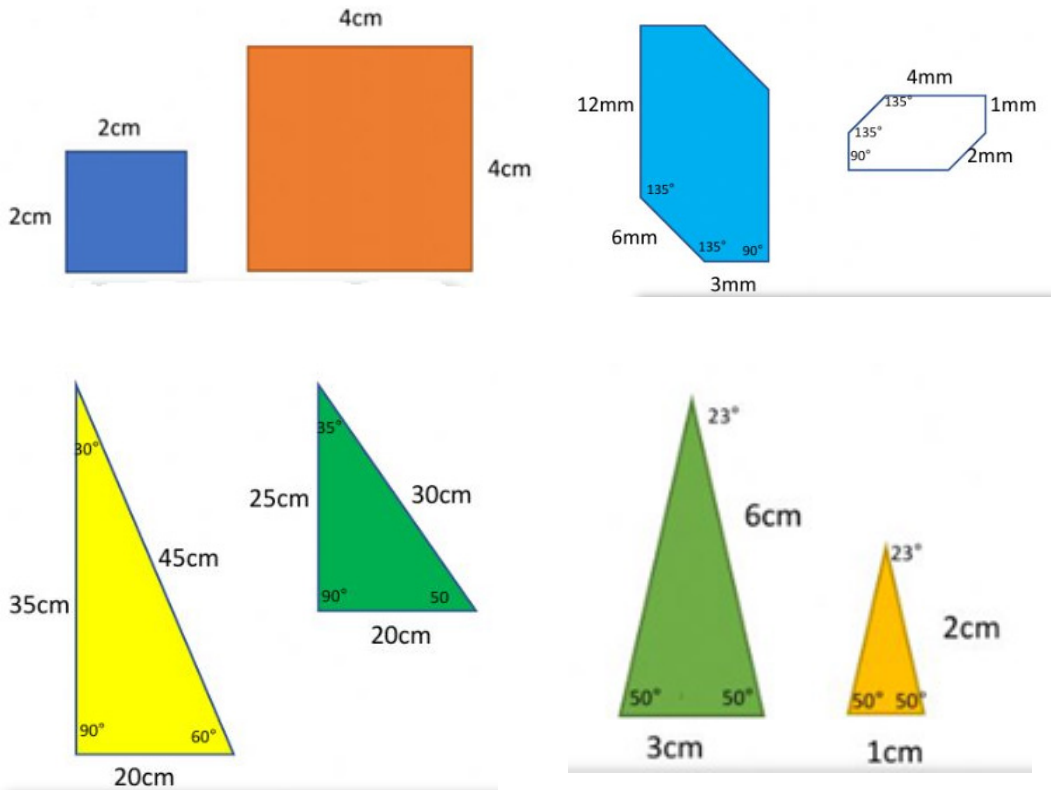
Step 1: Observe objects and group them according to their similarity and explain why you say that they are similar.



Expected answer:



Step 3: Observe each of the following cases, discuss the corresponding angles, the corresponding sides and determine the ratio of corresponding sides. Confirm which of them are similar



e) Results, interpretation, and conclusion

Two polygons are similar if:

- The ratio of corresponding sides is
- The corresponding angles are

Expected answers:

Two polygons are similar if:

1. The ratio of corresponding sides is constant,
2. The corresponding angles are equal.

PRACTICAL ACTIVITY/ EXPERIMENT 20: Explore Thales' theorem

a) Rationale:

This experiment is conducted when teaching lengths of sides of similar shapes using similarity and Thales theorem. It is taught in unit 11 of S3.

Thales theorem is used when arranging tiles and painting. It can be useful as paintings, tiles, and many other things as such that require precision to make sure that there is a proportional way.

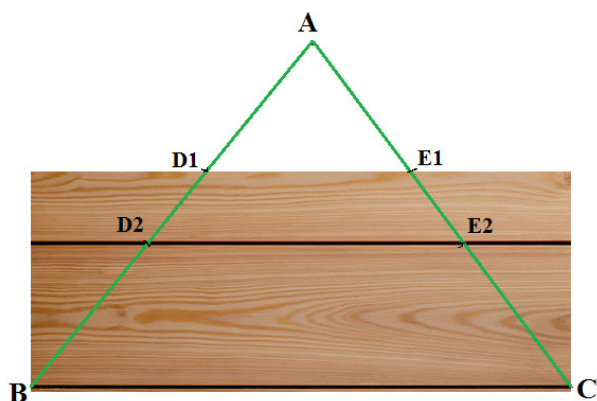
b) Objective

To verify the basic proportionality theorem, or Thales theorem that highlights that If a line is drawn parallel to one side of a triangle intersecting the other two sides then the line divides these sides in the same ratio.

c) Materials Required

Two well prepared timbers of different Lengths, nails, thread, ruler.

d) Procedure and diagram



Step 1: Place the two timbers one by another on the floor in the playground.

Step 2: Fix a nail above the two timbers on the point A.

Step 3: Fix 2 nails on the base of the first timber one in B and the other in C.

Step 4: Fix a thread joining the nail A and the nail in B. This thread passes by the upper base of the first timber on the point D2 and the upper base of the second timber on the point D1.

Step 5: Fix a thread joining the nail A and the nail in C. This thread passes by the upper base of the first timber on the point E2 and the upper base of the second timber on the point E1.

Step 6: Use the ruler and measure the lengths of the line segments AD1, D1B, AE1, E1C, AD2, D2B, AE2 and E2C, and record them in the observation table.

For each side measured on the transversal AB, what is its corresponding side on the transversal AC?

e) Recording data

| | | | | | |
|--------|--------|--------|--------|---------------------|---------------------|
| AD_1 | D_1B | AE_1 | E_1C | $\frac{AD_1}{D_1B}$ | $\frac{AE_1}{E_1C}$ |
| | | | | | |
| AD_2 | D_2B | AE_2 | E_2C | $\frac{AD_2}{D_2B}$ | $\frac{AE_2}{E_2C}$ |
| | | | | | |

f) Interpretation of Result and conclusion

How are lines BC and D₁E₁? How do we call lines AB and AC vis a vis the parallel lines BC and D₁E₁?

What can you say about the values $\frac{AD_1}{D_1B}$ and $\frac{AE_1}{E_1C}$?

Can you explain that $\frac{AD_2}{D_2B} = \frac{AE_2}{E_2C}$?

What is your Conclusion about the ratio of corresponding sides?

Expected answers:

The line D₁E₁ drawn parallel to one side BC of a triangle ABC intersecting the other two sides (AB and AC) divides these sides in the same ratio $\frac{AD_1}{D_1B} = \frac{AE_1}{E_1C}$.

In the same way, $\frac{AD_2}{D_2B} = \frac{AE_2}{E_2C}$ because the line D₂E₂ is parallel to the side BC and it intersects the sides AB and AC. In addition, this conclusion can be gotten when we consider that the triangle AD₁E₁, the triangle AD₂E₂ and the Triangle ABC are **similar**.

g) Guidance on the evaluation:

Ask students to consider the obtained diagram, measure the lengths of other corresponding sides and then determine the related ratio.

PRACTICAL ACTIVITY/ EXPERIMENT 21: Area of similar shapes

a) Rationale:

This experiment is done when teaching the similar polygons. It is taught in unit 11 of S3.

In real life, any form of tiling involves polygons. The tiles need to be similar or to tessellate to cover an area without leaving any gaps. This is directly connected to the angle properties of polygons. Architects include polygons with every plan of a house because rooms usually have 90° corners, but not always. When a designer has two similar objects, he can need to verify it by comparing their areas vis-a-vis the ratio of squares of their corresponding sides.

b) Objective:

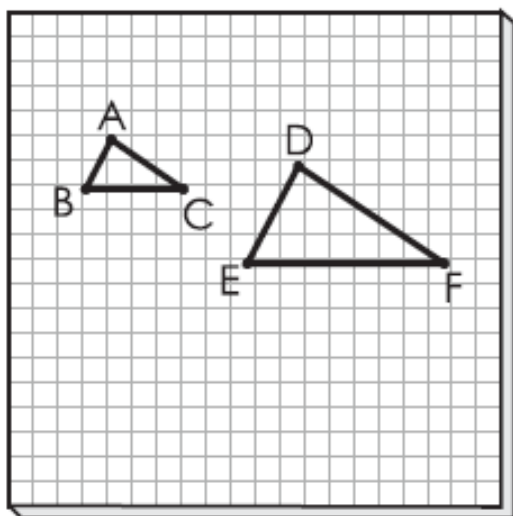
To verify that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides. It is done to show that if you have the ratio of areas of the similar polygons and one side of the polygon, it is easy to find the length of the side corresponding to the given side in the other polygon.

c) Required materials

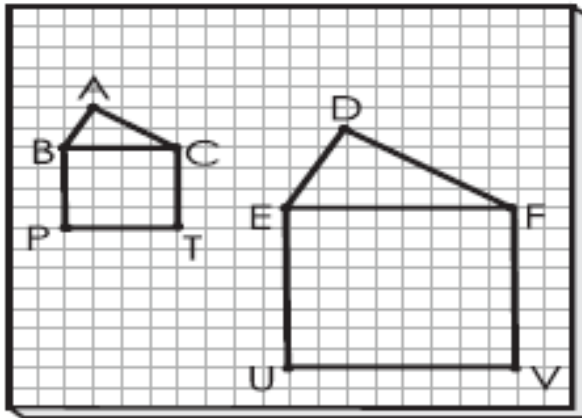
Geoboard, rubber bands, pencil, pen, ruler.

d) Procedures and Illustration

Step 1: Make two similar triangles ABC and DEF by considering 6 geoboard pins at suitable places on a geoboard and rubber band as illustrated in the following figure:



Step 2: Using rubber bands and the considered geoboard pins make squares on sides BC and EF as illustrated below:



Step 3: Find out the area of the two triangles and the two squares formed above by counting the number of unit squares enclosed in it in the following manner:

- Count one complete unit square enclosed by the figure as 1 and take its area as 1 square unit.
- Count the unit square which is more than half enclosed by the figure as 1 and take its area as 1 square unit.
- Count the unit square which is half enclosed by the figure as $1/2$ and take its area as $1/2$ square unit.
- Neglect the unit squares which are less than half enclosed by the figure.

Step 4: Also find out the length of sides BC and EF

Step 5: Repeat the above activity by forming different pairs of similar triangles by correctly suitably changing the geoboard pins and complete the following table:

(Use yes or no in the last column).

| No | $ar.tr(ABC)$ | $ar.tr(DEF)$ | $(BC)^2$ | $(EF)^2$ | $\frac{ar.tr(ABC)}{ar.tr(DEF)}$ | $\frac{(BC)^2}{(EF)^2}$ | $\frac{ar.tr(ABC)}{ar.tr(DEF)} = \frac{(BC)^2}{(EF)^2}$ |
|----|--------------|--------------|----------|----------|---------------------------------|-------------------------|---|
| 1 | | | | | | | |
| 2 | | | | | | | |
| 3 | | | | | | | |
| 4 | | | | | | | |

e) Results, interpretation, and conclusion

What is the expression that represents the ratio of areas of two similar triangles?

What is the expression that represents the ratio of squares of the corresponding sides?

What can you conclude basing on your answers in the last column?

Expected answer:

The ratio of areas of two similar triangles is $\frac{ar.tr(ABC)}{ar.tr(DEF)}$, ratio of squares of the corresponding sides is $\frac{(BC)^2}{(EF)^2}$. The two ratios are equal.

Therefore, the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

f) Guidance on the evaluation:

Ask students to consider the ratio of corresponding sides other than the one considered before and let them verify the equality.

PRACTICAL ACTIVITY/ EXPERIMENT 22: Image of an object under an enlargement

a) Rationale:

This experiment is conducted when teaching enlargement and its properties. It is taught in unit 11 of S3. Enlargement is a transformation by which the size of an object is increased. Real life examples of enlargements/reductions are: architectural perspective, scaled house plans and enlarging images on a computer.

b) Objectives:

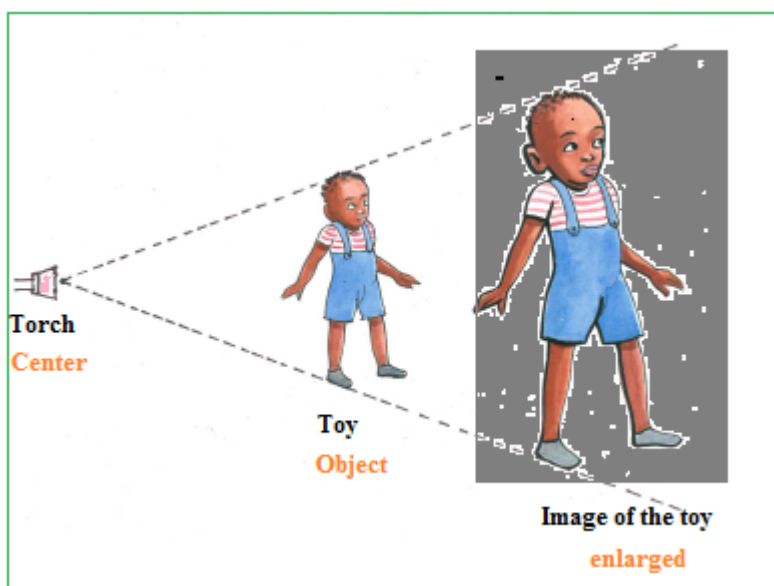
- Construct images of a shape by enlargement and compare the new shapes to their initial forms.
- Discuss the properties of enlargement used to transform these shapes to their new forms.

c) Material required:

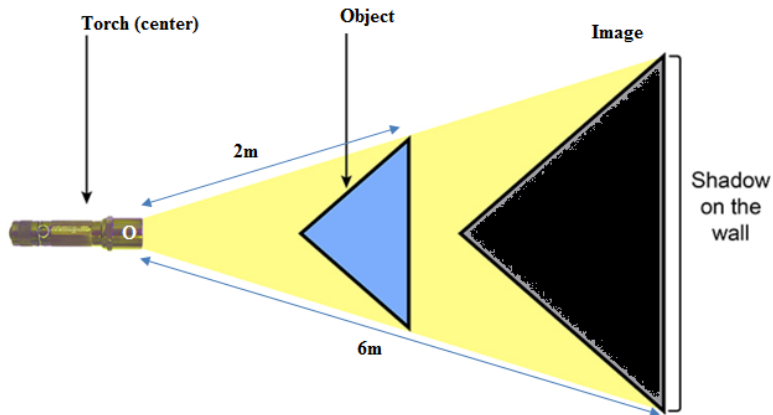
Toys of the same type but with different sizes, a triangular object, a rectangular object, thread, ruler, nails.

d) Procedure:

Step 1: Take a torch and put the toy in front of the torch at a short distance. What is the relationship between the toy object and the image (shadow) formed?



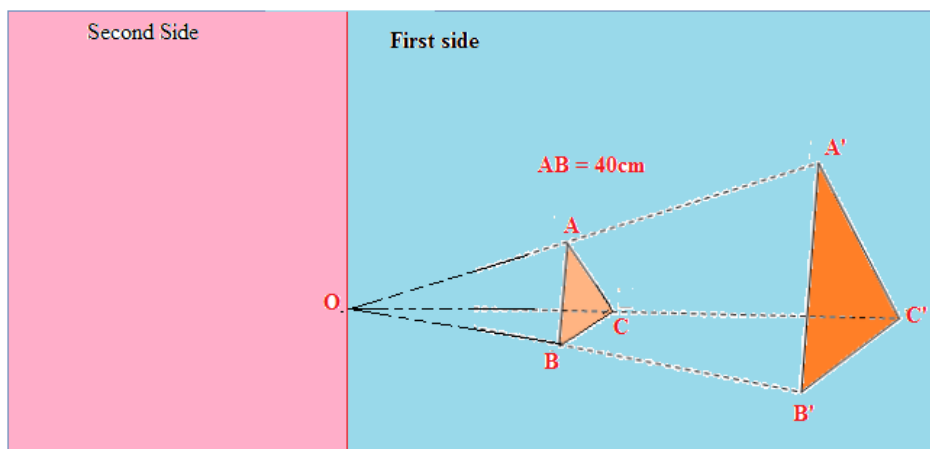
Step 2: Replace the toy by a triangular object such that the distance from the torch and the object is 2m.



Step 3: Now consider the triangular object ABC fixed on a manila paper and the point O considered as a center such that the distance $AB = 40\text{cm}$. Join each vertex of ΔABC (object points) to centre O with straight lines.

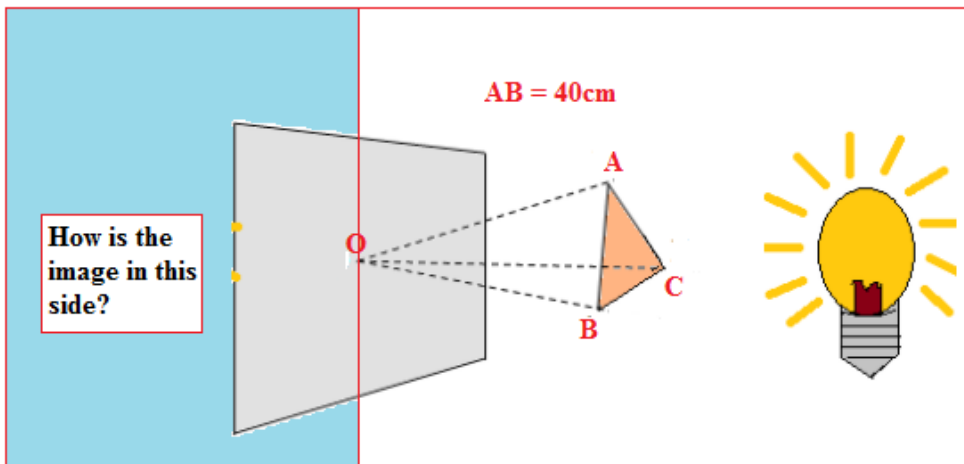
Step 4: Prolong each of these lines in the opposite side from centre O such that the distance between O and the image becomes 3 times the distance between the centre O and the vertex object.

Step 5: Join the images (new points) obtained to form the required image triangle $\Delta A'B'C'$ as illustrated here below:



Step 6: Discuss the characteristics of the image obtained in the first side.

Step 7: Repeat the same steps to find the image of the triangular object ABC in the second side and discuss the properties of the image obtained.



How is the image in this side?

e) Results, interpretation, and conclusion

Measure OA, OB and OC. Calculate the corresponding lengths OA', OB' and OC'; and complete the following table:

| OA | OA' | OB | OB' | OC | OC' | OA'/OA | OB'/OB | OC'/OC |
|----|-----|----|-----|----|-----|--------|--------|--------|
| | | | | | | | | |

Is the ratio in the last column the same? How do you call it? Define this ratio.

Discuss the position of the image vis a-vis the center and the object.

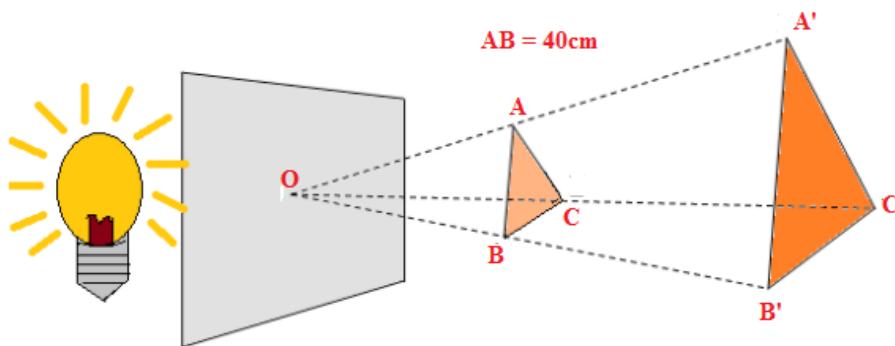
Expected answer:

We see that image distance = $k \times$ object distance, where k is the scale factor.

Thus, $OA' = 3OA$, $OB' = 3OB$ and $OC' = 3OC$.

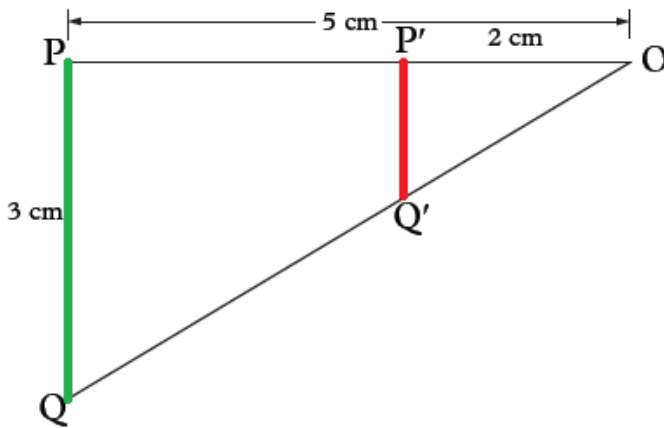
This means that $A'B'C'D'$ is the image of $ABCD$ under enlargement with centre O and scale factor 3.

- If the scale factor is positive, the object and the image are on the same side of the centre of enlargement. In real life this Enlargement (resizing of object) is called dilation or expansion (getting bigger).

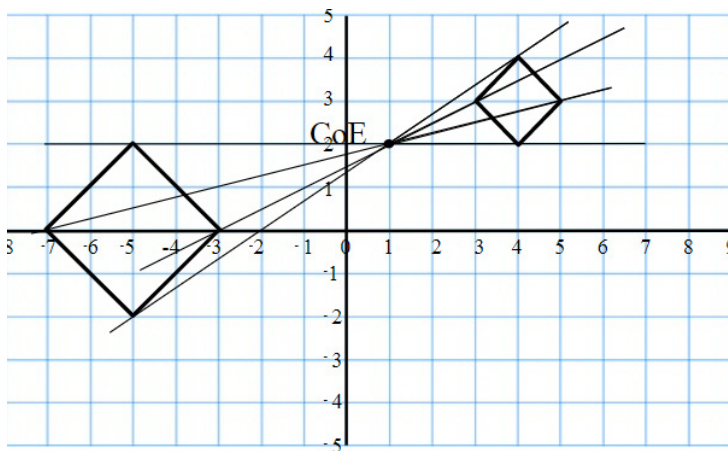


- If the scale factor is greater than 1, the image is greater than the object. If the scale factor is a simple fraction (between 0 and 1), the image is smaller than

the object and lies between the object and the centre of enlargement. This type of enlargement is called **Reduction, Contraction or Compression (reducing the size of an object)**.

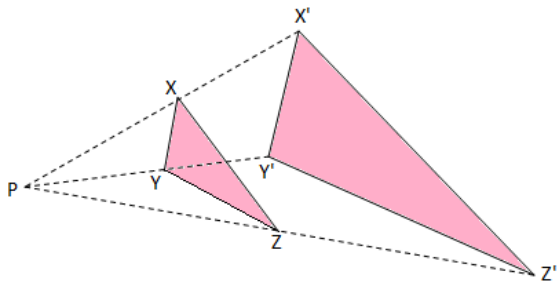


- Under enlargement with a **negative scale factor**,
 1. The object and the image are on **opposite sides** of the centre of enlargement,
 2. The image is larger or smaller than the object depending on whether the scale factor is greater than 1 and negative or a negative proper fraction:



f) Guidance on the evaluation:

Give students an related activity to be done to find the image of an object. For example: A triangular object XYZ has been enlarged to $\Delta X'Y'Z'$ about the centre of enlargement O by a scale factor 2.5.



Find:

- (i) $X'Y'$ if $XY = 4$ cm,
- (ii) XZ if $X'Z' = 15$ cm.

Expected answer:

i) $k = \frac{X'Y'}{XY}$ or $\frac{X'Y'}{4\text{cm}} = 2.5$. Therefore, $X'Y' = 2.5 \times (4\text{cm}) = 10$ cm

ii) $k = \frac{X'Z'}{XZ}$ or $\frac{15\text{cm}}{XZ} = 2.5$. Therefore, $XZ = \frac{15\text{cm}}{2.5} = 6$ cm

PRACTICAL ACTIVITY/ EXPERIMENT 23: Scale factor for an enlargement

a) Rationale:

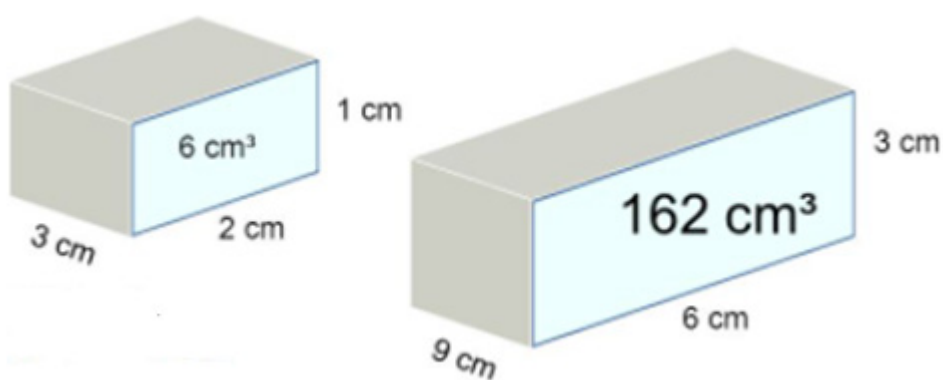
This experiment is conducted when teaching the enlargement. It is taught in unit 11. It can help students to be able to make an object which is bigger than the one they have but with the same shape.

b) Objective:

Find the linear scale factor and centre of an enlargement.

c) Required material:

Ruler, thread, scotch, pair of compasses, two boxes with different sizes such that the ratio of corresponding edges is the same (see the figure below):

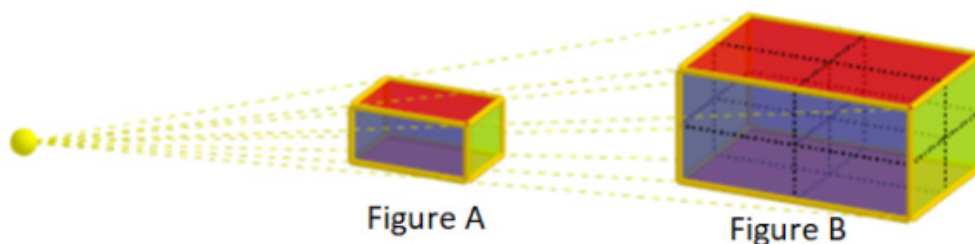


d) Procedure:

Step 1: Measure the length of corresponding edges.

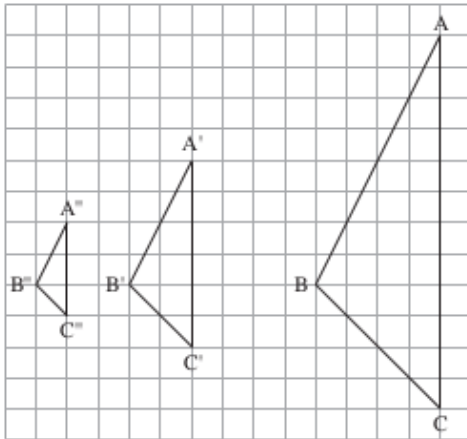
Step 2: Calculate the ratio of corresponding edges and conclude if it is an enlargement.

Step 3: Use 2 or 3 threads to join corresponding vertices and guess where threads can meet. (see the figure below)



How do we call the point C the intersection of lines joining the corresponding edges (intersecting point of threads)?

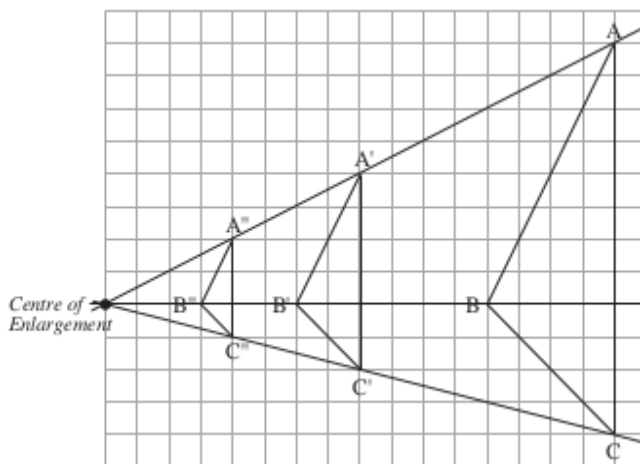
Step 4: The diagram shows three triangles. ABC was enlarged with different scale factors to give $A'B'C'$ and $A''B''C''$. Discuss how to find the centre of enlargement and the scale factor for each enlargement



Complete the following table and give the scale factor for : the enlargement that reduces ABC to $A'B'C'$, and the scale factor for the enlargement that reduces ABC to $A''B''C''$.

| AC | $A'C'$ | $A''C''$ | $AC/A'C'$ | $AC/A''C''$ |
|----|--------|----------|-----------|-------------|
| | | | | |
| AB | $A'B'$ | $A''B''$ | $AB/A'B'$ | $AB/A''B''$ |
| | | | | |

Expected answer:



e) Interpretation of results and conclusion:

When the object and the image under an enlargement are given,
What is necessary to find its center?

Complete this sentence: An enlargement is described if we have and

Expected answers:

- Given an **object** and its **image**, it is sufficient to construct only two lines to obtain the centre of enlargement.
- An enlargement is fully described if the **centre and the scale factor** are given.

PRACTICAL ACTIVITY 24: Explore the correlation between two variables

a) Rationale:

This activity is conducted when teaching the bivariate data. It is taught in unit 13. It can help students to be able to understand the meaning of bivariate data, and the correlation between two variables.

b) Objective:

Represent bivariate data in a scatter diagram and discuss the type of correlation between two variables.

c) Required material:

Ruler, large grided sheet of paper, pens.

d) procedure:

Step 1: Discuss the meaning of bivariate data,

Step 2: Read the following problem:

The experiment was conducted to explore the relationship between the height of fathers and that of their sons. The table below shows data recorded from 10 men who brought their children at a health centre. The height of fathers and their respective sons are given in the following table:



Each son is taller than his father

| No | Height of father | Height of boys |
|----|------------------|----------------|
| 1 | 164 | 165 |
| 2 | 166 | 167 |
| 3 | 150 | 151 |
| 4 | 162 | 164 |
| 5 | 159 | 160 |
| 6 | 165 | 167 |
| 7 | 187 | 188 |

| | | |
|----|-----|-----|
| 8 | 152 | 154 |
| 9 | 168 | 170 |
| 10 | 180 | 181 |

Step 3: Get the large grided paper and answer questions:

- a) draw a cartesian plane and plot recorded data.
- c) show the scatter diagram for the data
- d) Find the equation of line passing by points (164,165) and (166,167).
 - i) is the obtained line a line of the best fit?
 - ii) Is it correct to use the obtained line to find the height of the son whose father is 200cm tall?
- e) Are the two variables (the height of fathers and the height of their sons) correlated? Which type of correlation is between the two variables?

e) Results, interpretation, and conclusion

The **scatter diagram** is a graph that shows how the points in the graph are scattered along an underlying linear relation between the variables rather than falling on it.

In a situation where one variable positively affects another variable, we say **positive correlation** has occurred. The diagram shows that the increase on one variable gives the increase of the other.

When one variable affects another variable negatively, we say **negative correlation** has occurred. In this case, the increase of one variable gives the decrease of the other.

When the increase of one variable does not affect the other variable, we say that there is **no correlation** between them.

f) Guidance on the evaluation

Give students an activity to be done. For example: From a laboratory research, data were collected on mass of rabbits and mass of their hearts and recorded data in the following table:

| | | | | | |
|----------------------------|-------------|-------------|-------------|-------------|-------------|
| Mass of rabbit (kg) | 1.22 | 1.54 | 1.26 | 1.19 | 1.23 |
| Mass of heart (mg) | 772 | 837 | 761 | 910 | 691 |

Use the table to:




- (a) Construct a scatter diagram for the data.
- (b) Estimate a line of best fit.



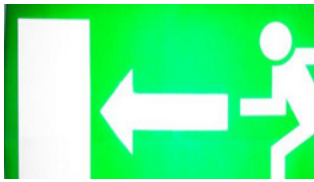
- (c) Find the equation of the line.
- (d) How can you use the equation in c to make predictions?
- (e) Describe the correlation in this data.



REFERENCE

- Ministry of Education and REB (2015). Mathematics Syllabus for Ordinary Level (S1-S3), Kigali, Rwanda.
- UR-CE (2020) Continuous professional development certificate in innovative teaching mathematics and science (CPD ITMS), Module 1: Innovative Teaching methods for Mathematics and Science (CMS1141), Kigali: UR-CE.
- Ministry of Education and REB (2020). Mathematics S1 Student's book, Kigali, Rwanda.
- Ministry of Education and REB (2020). Mathematics S2 Student's book, Kigali, Rwanda.
- Ministry of Education and REB (2020). Mathematics S3 Student's book, Kigali, Rwanda.

Annex 1: Name of commonly hazard symbols useful in the laboratory

| S/N | Name | Hazard Symbol | Explanation |
|-----|---------------------------|---|--|
| 1 | Flammable and combustible |  | The flammable and combustible symbol signifies substances that will ignite and continue to burn in air. Substances in this category may be gases, aerosols, liquids, or solids, and include many solvents and cleaning materials that are commonly used in the laboratory. |
| 2 | Oxidizing agents |  | <p>The symbol for oxidizing materials indicates the presence of chemicals that readily give off oxygen or other oxidizing substances.</p> <p>Oxidizing materials may intensify fires and cause explosions, and also may be toxic or corrosive.</p> <p>Some common oxidizing liquids and solids found in laboratories are bromine, chlorates, nitrates, perchloric acid, and peroxides.</p> |
| 3 | Toxic |  | <p>A substance known to pose that is classified as posing skin corrosion or irritation; serious eye damage or eye irritation; respiratory or skin sensitization; germ cell mutagenicity; carcinogenicity; reproductive toxicity, and other toxicity is classified as hazardous or toxic substance.</p> <p>These substances can cause death or damage to health by inhalation, ingestion or skin absorption.</p> <p>Example: acid</p> |

| | | | |
|---|----------------|---|---|
| 4 | Irritants |  | <p>Irritants are substances that cause reversible inflammatory effects on living tissue at the site of contact.</p> |
| 5 | Magnetic Field |  | <p>Certain pieces of laboratory equipment generate strong magnetic fields. The strong magnetic field sign alerts lab members to the dangers that this type of equipment can pose.</p> <p>The risks are especially imminent for people wearing pacemakers and implants, which will tend to align themselves with the magnetic field lines, as will watches, clipboards, and certain tools.</p> <p>Magnetic fields result from the flow of current through wires or electrical devices.</p> <p>Examples of sources: machines, electrical wiring (such as power lines)</p> |
| 6 | Exit |  | <p>It is good to know where all of the exits are located, especially when working in a laboratory environment where you may need to get out quickly.</p> <p>Labs are required to mark exits routes from the area with clearly identifiable signs.</p> |

| | | | |
|---|-------------------|---|---|
| 7 | Fire extinguisher |  | <p>Fires can happen anywhere, but lab fires can be even more dangerous due to Bunsen burners, flammable liquids, research documents, laptops, and lab equipment that might be present at any given time.</p> <p>It is essential that the occupants of a laboratory are fully aware of the risks and the appropriate extinguishing media. A fire extinguisher safety sign indicates the exact location of a lab's fire extinguisher.</p> |
| 8 | Electrical hazard |  | <p>The electrical hazard safety symbol, which typically includes a frayed wire and a hand with a lightning bolt across it, indicates any electrical hazards in the lab.</p> <p>If an electrical hazard is suspected, the device in question should be disconnected immediately and the cause determined by a qualified technician.</p> <p>Equipment should always be turned off and unplugged when any work is being done on it.</p> |

